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71. (2), (3) 展開 (4)  $\frac{3x^4 - 2x + 1}{x^2} = \frac{3x^4}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2} = 3x^2 - \frac{2}{x} + x^{-2}$

(5)  $\sqrt{x}(x+2) = \sqrt{x}x + 2\sqrt{x} = x^{\frac{3}{2}} + x^{\frac{1}{2}}$  (6)  $\sqrt[4]{x^3} + \frac{1}{\sqrt[4]{x^3}} = x^{\frac{3}{4}} + x^{-\frac{3}{4}}$

72. (2)  $\sin^2 x = 1 - \cos^2 x \therefore \frac{\sin^2 x}{1 - \cos x} = \frac{1 - \cos^2 x}{1 - \cos x} = \frac{(1 + \cos x)(1 - \cos x)}{1 - \cos x} = 1 + \cos x$

(3)  $\frac{1 + \cos^3 x}{\cos^2 x} = \frac{1}{\cos^2 x} + \cos x$  (4)  $\frac{1}{\sqrt{4-4x^2}} = \frac{1}{\sqrt{4}\sqrt{1-x^2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{1-x^2}}$

73. (1), (2), (4), (5) 展開 (3)  $\frac{x^2 - 3}{x} = x - \frac{3}{x}$  (7) 半角の公式より  $\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$

(8) 2倍角の公式より  $\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$

74. 偶関数, 奇関数の積分の性質を用いる. (3)  $\cos x$  は偶関数,  $\sin x$  は奇関数だから

与式  $= 2 \int_0^{\frac{\pi}{2}} 3 \cos x dx$  (4)  $\frac{x+1}{x^2+1} = \frac{x}{x^2+1} + \frac{1}{x^2+1}$  ここで  $\frac{x}{x^2+1}$  は奇関数,  $\frac{1}{x^2+1}$  は偶関数.

75. (2) (1) より  $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{\cos^2 x \sin^2 x} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left( \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx$

76.  $\int_{-1}^3 f(x) dx = \int_{-1}^1 f(x) dx + \int_1^3 f(x) dx.$

$f(x)$  の定義より 与式  $= \int_{-1}^1 (-x^2 + 4x) dx + \int_1^3 (x^2 - 2x + 4) dx$

77.  $f(x) = ax^2 + bx + c$  とおくと  $f'(x) = 2ax + b$ .  $f(1) = 1$ ,  $f'(1) = 4$  より 2つの式をえる.

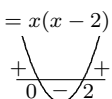
$\int_{-1}^1 f(x) dx = \int_{-1}^1 (ax^2 + bx + c) dx = a \int_{-1}^1 x^2 dx + b \int_{-1}^1 x dx + c \int_{-1}^1 dx = 4$

$x^2, 1$  は偶関数  $x$  は奇関数だから  $2a \int_0^1 x^2 dx + 2c \int_{-1}^1 dx = 4$

78. 偶関数, 奇関数の積分の性質を用いる  $\int_{-1}^1 (ax^3 + bx^2 + cx + d) dx =$

$a \int_{-1}^1 x^3 dx + b \int_{-1}^1 x^2 dx + c \int_{-1}^1 x dx + d \int_{-1}^1 dx = 2b \int_0^1 x^2 dx + 2d \int_{-1}^1 dx$  他も同様.

79. (1)  $|x| = \begin{cases} x & (x \geq 0) \\ -x & (x < 0) \end{cases}$  より  $\int_{-2}^1 |x| dx = \int_{-2}^0 |x| dx + \int_0^1 |x| dx = \int_{-2}^0 (-x) dx + \int_0^1 x dx$

$y = x(x-2)$   


(2) も同様右グラフより  $x \leq 0, 2 \leq x$  のとき  $x(x-2) \geq 0 \therefore |x(x-2)| = x(x-2),$

$0 < x < 2$  のとき  $x(x-2) < 0 \therefore |x(x-2)| = -x(x-2).$

80. (1)  $F(x) = \int_1^x \sqrt{t} \log t dt$  とおくと  $F(x^2) = \int_1^{x^2} \sqrt{t} \log t dt$

$\therefore \{F(x^2)\}' = F'(x^2) \cdot (x^2)' = 2xF'(x^2)$ . 微分積分法の基本定理より  $F'(x) = \sqrt{x} \log x$  だから

$\{F(x^2)\}' = 2x\sqrt{x^2} \log x^2 = \dots$

(2)  $F(x) = \int_0^x t^3 e^t dt$  とおくと

$\int_{-x}^x t^3 e^t dt = \int_{-x}^0 t^3 e^t dt + \int_0^x t^3 e^t dt = - \int_0^{-x} t^3 e^t dt + \int_0^x t^3 e^t dt = -F(-x) + F(x)$

$$\left\{ \int_{-x}^x t^3 e^t dt \right\}' = -\{F(-x)\}' + F'(x) = -F'(-x) \cdot (-x)' + F'(x) = F'(-x) + F'(x).$$

微分積分の基本定理より  $F'(x) = x^3 e^x$  だから  $\left\{ \int_{-x}^x t^3 e^t dt \right\}' = (-x)^3 e^{-x} + x^3 e^x = \dots$

81. (1)  $\int_0^{\frac{\pi}{2}} f(t) dt = c \dots \textcircled{1}$  とおくと  $f(x) = \cos x + c \dots \textcircled{2} \therefore f(t) = \cos t + c.$

これを①に代入.  $\int_0^{\frac{\pi}{2}} (\cos t + c) dt = c$

$$\text{左辺} = [\sin t + ct]_0^{\frac{\pi}{2}} = \sin \frac{\pi}{2} - \sin 0 + c \cdot \frac{\pi}{2} = 1 + \frac{\pi}{2}c$$

(2)  $f(x) = 1 + \int_{-1}^1 x^2 f(t) dt = 1 + x^2 \int_{-1}^1 f(t) dt. \int_{-1}^1 f(t) dt = c \dots \textcircled{1}$  とおくと

$f(x) = 1 + cx^2 \dots \textcircled{2} \therefore f(t) = 1 + ct^2$  これを①に代入. 以下 (1) と同様

82. (1)  $0 \leq x \leq \frac{1}{\sqrt{2}}$  のとき  $0 \leq x^4 \leq x^2 \therefore 0 \geq -x^4 \geq -x^2 \therefore 1 \geq 1 - x^4 \geq 1 - x^2$

$$\text{よって } 1 \geq \sqrt{1 - x^4} \geq \sqrt{1 - x^2} \therefore 1 \leq \frac{1}{\sqrt{1 - x^4}} \leq \frac{1}{\sqrt{1 - x^2}}$$

(2) (1) の各辺を 0 から  $\frac{1}{\sqrt{2}}$  まで積分すると  $\int_0^{\frac{1}{\sqrt{2}}} dx \leq \int_0^{\frac{1}{\sqrt{2}}} \frac{dx}{\sqrt{1 - x^4}} \leq \int_0^{\frac{1}{\sqrt{2}}} \frac{dx}{\sqrt{1 - x^2}}$

$$\text{ここで } \int_0^{\frac{1}{\sqrt{2}}} \frac{dx}{\sqrt{1 - x^2}} = [\text{Sin}^{-1} x]_0^{\frac{1}{\sqrt{2}}}$$