

119. (1) $t = 0$ のとき $x = 0, y = 1$. $t = 1$ のとき $x = 1, y = 0$. よって曲線と x 軸, y 軸で囲まれる.

$$S = \int_0^1 \left| y \frac{dx}{dt} \right| dt = \int_0^1 |(t^2 - 2t + 1)(t^2)'| dt = \int_0^1 (t^2 - 2t + 1)2t dt = \dots$$

(2) $t = 0$ のとき $x = 0, y = 1$. $t = \frac{\pi}{4}$ のとき $x = 1, y = \frac{\sqrt{2}}{2} + 1$. よって曲線と x 軸, y 軸, 直線 $x = 1$ で囲まれる.

$$S = \int_0^{\frac{\pi}{4}} \left| y \frac{dx}{dt} \right| dt = \int_0^{\frac{\pi}{4}} |(\sin t + 1)(\tan t)'| dt = \int_0^{\frac{\pi}{4}} \frac{\sin t + 1}{\cos^2 t} dt = \int_0^{\frac{\pi}{4}} \frac{\sin t}{\cos^2 t} dt + \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 t} dt.$$

1 番目の積分は $\cos t = u$ とおく.

(3) $t = 0$ のとき $x = 1, y = 2$. $t = \frac{\pi}{2}$ のとき $x = 0, y = 0$. よって曲線と x 軸, 直線 $x = 1$ で囲まれる.

$$S = \int_0^{\frac{\pi}{2}} \left| y \frac{dx}{dt} \right| dt = \int_0^{\frac{\pi}{2}} |(\cos 2t + 1)(\cos t)'| dt = \int_0^{\frac{\pi}{2}} (\cos 2t + 1) \sin t dt.$$

半角の公式より $\frac{1 + \cos 2t}{2} = \cos^2 t$. $\therefore \cos 2t + 1 = 2 \cos^2 t$ と変形してから $\cos t = u$ とおく.

$$120. (1) \frac{dx}{dt} = (3t^2)' = 6t, \frac{dy}{dt} = (3t - t^3)' = 3 - 3t^2. \text{ よって } l = \int_0^{\sqrt{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \\ = \int_0^{\sqrt{3}} \sqrt{(6t)^2 + (3 - 3t^2)^2} dt = \int_0^{\sqrt{3}} \sqrt{9t^4 + 18t^2 + 9} dt = \int_0^{\sqrt{3}} 3(t^2 + 1) dt = \dots$$

(2) $\frac{dx}{dt} = (\cos t + t \sin t)' = -\sin t + \sin t + t \cos t = t \cos t, \frac{dy}{dt} = (\sin t - t \cos t)' = \cos t - \cos t + t \sin t = t \sin t.$

$$\text{よって } l = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^{\pi} \sqrt{(t \cos t)^2 + (t \sin t)^2} dt = \int_0^{\pi} \sqrt{t^2(\cos^2 t + \sin^2 t)} dt = \int_0^{\pi} t dt.$$

(3) $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (-2 \sin t + 2 \sin 2t)^2 + (2 \cos t - 2 \cos 2t)^2$

$$= 4 \sin^2 t - 8 \sin t \sin 2t + 4 \sin^2 2t + 4 \cos^2 t - 8 \cos t \cos 2t + 4 \cos^2 2t$$

$$= 4(\sin^2 t + \cos^2 t) - 8(\sin t \sin 2t + \cos t \cos 2t) + 4(\sin^2 2t + \cos^2 2t) = 4 - 8 \cos(2t - t) + 4 = 8 - 8 \cos t.$$

(第 2 項の変形には加法定理を用いた.) 半角の定理より $1 - \cos t = 2 \sin^2 \frac{t}{2}$ だから

$$l = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^{\pi} \sqrt{8 \cdot 2 \sin^2 \frac{t}{2}} dt = \int_0^{\pi} 4 \sin \frac{t}{2} dt = \left[-8 \cos \frac{t}{2} \right]_0^{\pi}$$

$$121. (1) \frac{dx}{dt} = (t^{\frac{1}{2}})' = \frac{1}{2} t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}. \therefore V = \pi \int_0^1 y^2 \left| \frac{dx}{dt} \right| dt = \pi \int_0^1 (\sqrt{t} - t)^2 \frac{1}{2\sqrt{t}} dt = \pi \int_0^1 \frac{t - 2t\sqrt{t} + t^2}{2\sqrt{t}} dt \\ = \frac{1}{2} \pi \int_0^1 (\sqrt{t} - 2t + t\sqrt{t}) dt$$

$$(2) \therefore V = \pi \int_0^{\frac{\pi}{2}} y^2 \left| \frac{dx}{dt} \right| dt = \pi \int_0^{\frac{\pi}{2}} \sin^2 2t \cos t dt = \pi \int_0^{\frac{\pi}{2}} (2 \sin t \cos t)^2 \cos t dt = 4\pi \int_0^{\frac{\pi}{2}} \sin^2 t \cos^3 t dt$$

(2 倍角の公式 $\sin 2t = 2 \sin t \cos t$ を用いた) $\cos^3 t$ のうち $\cos^2 t = 1 - \sin^2 t$ と置き換えて $\sin t = u$ とおく.

$$122. (1) S = 2\pi \int_0^{\frac{\pi}{2}} |y| \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = 2\pi \int_0^{\frac{\pi}{2}} 3 \sin t \sqrt{\{(\cos 2t)'\}^2 + \{(3 \sin t)'\}^2} dt \\ = 2\pi \int_0^{\frac{\pi}{2}} 3 \sin t \sqrt{(-2 \sin 2t)^2 + (3 \cos t)^2} dt = 6\pi \int_0^{\frac{\pi}{2}} \sin t \sqrt{16 \sin^2 t \cos^2 t + 9 \cos^2 t} dt \\ = 6\pi \int_0^{\frac{\pi}{2}} \sin t \sqrt{16 \sin^2 t + 9} \cos t dt \quad (2 \text{ 倍角の公式 } \sin 2t = 2 \sin t \cos t \text{ を用いた)}$$

$$16 \sin^2 t + 9 = u \text{ とおくと } 32 \sin t \cos t dt = du. \quad \begin{array}{|c|c|c|} \hline t & 0 & \rightarrow \frac{\pi}{2} \\ \hline u & 9 & \rightarrow 25 \\ \hline \end{array}$$

$$\therefore V = 6\pi \int_9^{25} \sqrt{u} \frac{1}{32} du$$

$$(2) \frac{dx}{dt} = (\cos t \sqrt{\cos 2t})' = -\sin t \sqrt{\cos 2t} + \cos t \cdot \frac{1}{2} (\cos 2t)^{-\frac{1}{2}} (\cos 2t)' = -\sin t \sqrt{\cos 2t} + \frac{-2 \cos t \sin 2t}{2\sqrt{\cos 2t}}$$

$$= -\frac{\sin t \cos 2t + \cos t \sin 2t}{\sqrt{\cos 2t}} = -\frac{\sin(t+2t)}{\sqrt{\cos 2t}} = -\frac{\sin 3t}{\sqrt{\cos 2t}} \quad (\text{加法定理を用いた})$$

$$\frac{dy}{dt} = (\sin t \sqrt{\cos 2t})' = \cos t \sqrt{\cos 2t} + \sin t \cdot \frac{1}{2} (\cos 2t)^{-\frac{1}{2}} (\cos 2t)' = \cos t \sqrt{\cos 2t} + \frac{-2 \sin t \sin 2t}{2\sqrt{\cos 2t}}$$

$$= \frac{\cos t \cos 2t - \sin t \sin 2t}{\sqrt{\cos 2t}} = \frac{\cos(t+2t)}{\sqrt{\cos 2t}} = \frac{\cos 3t}{\sqrt{\cos 2t}} \quad (\text{加法定理を用いた})$$

$$\therefore V = 2\pi \int_0^{\frac{\pi}{4}} \sin t \sqrt{\cos 2t} \sqrt{\left(-\frac{\sin 3t}{\sqrt{\cos 2t}}\right)^2 + \left(\frac{\cos 3t}{\sqrt{\cos 2t}}\right)^2} dt = 2\pi \int_0^{\frac{\pi}{4}} \sin t \sqrt{\cos 2t} \sqrt{\frac{\sin^2 3t + \cos^2 3t}{\cos 2t}} dt.$$

$$= 2\pi \int_0^{\frac{\pi}{4}} \sin t \sqrt{\cos 2t} \frac{1}{\sqrt{\cos 2t}} dt = 2\pi \int_0^{\frac{\pi}{4}} \sin t dt.$$

$$123. (1) S = \frac{1}{2} \int_0^{\frac{\pi}{2}} r^2 d\theta = \frac{1}{2} \int_0^{\frac{\pi}{2}} \sin^2 2\theta d\theta = \frac{1}{2} \int_0^{\frac{\pi}{2}} 4 \sin^2 \theta \cos^2 \theta d\theta = 2 \int_0^{\frac{\pi}{2}} \sin^2 \theta (1 - \sin^2 \theta) d\theta$$

$$= 2 \left(\int_0^{\frac{\pi}{2}} \sin^2 \theta d\theta - \int_0^{\frac{\pi}{2}} \sin^4 \theta d\theta \right) = 2 \left(\frac{1}{2} \frac{\pi}{2} - \frac{3}{4} \frac{1}{2} \frac{\pi}{2} \right) = \frac{\pi}{8}$$

$$(2) S = \frac{1}{2} \int_0^{2\pi} (\cos \theta + 2)^2 d\theta = \frac{1}{2} \int_0^{2\pi} (\cos^2 \theta + 4 \cos \theta + 4) d\theta = \frac{1}{2} \int_0^{2\pi} \left(\frac{1 + \cos 2\theta}{2} + 4 \cos \theta + 4 \right) d\theta.$$

$$124. (1) l = \int_0^{\frac{\pi}{2}} \sqrt{r^2 + (r')^2} d\theta = \int_0^{\frac{\pi}{2}} \sqrt{e^{2\theta} + e^{2\theta}} d\theta = \int_0^{\frac{\pi}{2}} \sqrt{2} e^{\theta} d\theta.$$

$$(2) l = \int_0^{\frac{\pi}{2}} \sqrt{(\cos^2 \theta)^2 + (-2 \cos \theta \sin \theta)^2} d\theta = \int_0^{\frac{\pi}{2}} \sqrt{\cos^4 \theta + 4 \cos^2 \theta \sin^2 \theta} d\theta = \int_0^{\frac{\pi}{2}} \sqrt{\cos^2 \theta + 4 \sin^2 \theta} \cos \theta d\theta.$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{1 + 3 \sin^2 \theta} \cos \theta d\theta. \quad \sin \theta = t \text{ とおくと}$$

$$l = \int_0^1 \sqrt{1 + 3t^2} dt = \int_0^1 \sqrt{3} \sqrt{\frac{1}{3} + t^2} dt = \frac{\sqrt{3}}{2} \left[t \sqrt{t^2 + \frac{1}{3}} + \frac{1}{3} \log \left| t + \sqrt{t^2 + \frac{1}{3}} \right| \right]_0^1$$

$$= \frac{\sqrt{3}}{2} \left\{ \sqrt{\frac{4}{3}} + \frac{1}{3} \log \left(1 + \sqrt{\frac{4}{3}} \right) - \frac{1}{3} \log \sqrt{\frac{1}{3}} \right\} = \frac{\sqrt{3}}{2} \left\{ \frac{2}{\sqrt{3}} + \frac{1}{3} \log \left(1 + \frac{2}{\sqrt{3}} \right) - \frac{1}{3} \log \frac{1}{\sqrt{3}} \right\}$$

$$= 1 + \frac{\sqrt{3}}{6} \log \frac{1 + \frac{2}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = 1 + \frac{\sqrt{3}}{6} \log(\sqrt{3} + 2)$$

$$125. (1) \int_0^2 |e^{-t}| dt = \int_0^2 e^{-t} dt. \quad (2) \int_1^2 |2 \sin \pi t| dt = \int_1^2 (-2 \sin \pi t) dt.$$

$$126. (1) \text{与式} = \lim_{\varepsilon \rightarrow +0} \int_{\varepsilon}^2 \frac{1}{x^3} dx = \lim_{\varepsilon \rightarrow +0} \int_{\varepsilon}^2 x^{-3} dx = \lim_{\varepsilon \rightarrow +0} \left[-\frac{1}{2} x^{-2} \right]_{\varepsilon}^2 = \lim_{\varepsilon \rightarrow +0} \frac{1}{2} \left(-\frac{1}{4} + \frac{1}{\varepsilon^2} \right) = \infty. \text{よって存在しない.}$$

$$(2) \text{与式} = \lim_{\varepsilon \rightarrow +0, \varepsilon' \rightarrow +0} \int_{\varepsilon}^{1-\varepsilon'} \frac{dx}{\sqrt{x(1-x)}}. \quad \int \frac{dx}{\sqrt{x(1-x)}} = \int \frac{dx}{\sqrt{\frac{1}{4} - (x - \frac{1}{2})^2}}. \quad x - \frac{1}{2} = t \text{ とおくと}$$

$$\int \frac{dx}{\sqrt{x(1-x)}} = \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} = \text{Sin}^{-1} \frac{t}{\frac{1}{2}} = \text{Sin}^{-1} 2t = \text{Sin}^{-1}(2x - 1). \text{よって}$$

$$\text{与式} = \lim_{\varepsilon \rightarrow +0, \varepsilon' \rightarrow +0} [\text{Sin}^{-1}(2x - 1)]_{\varepsilon}^{1-\varepsilon'} = \lim_{\varepsilon \rightarrow +0, \varepsilon' \rightarrow +0} \{ \text{Sin}^{-1}(1 - 2\varepsilon') - \text{Sin}^{-1}(2\varepsilon - 1) \}$$

$$= \text{Sin}^{-1} 1 - \text{Sin}^{-1}(-1) = \frac{\pi}{2} - \left(\frac{\pi}{2} \right) = \pi.$$

$$(3) \text{与式} = \lim_{b \rightarrow \infty} \int_0^b x e^{-x} dx = \lim_{b \rightarrow \infty} \left\{ [-x e^{-x}]_0^b - \int_0^b (-e^{-x}) dx \right\} = \lim_{b \rightarrow \infty} (-b e^{-b} - [e^{-x}]_0^b) = \lim_{b \rightarrow \infty} (-b e^{-b} - e^{-b} + 1).$$

$$\lim_{b \rightarrow \infty} e^{-b} = \lim_{b \rightarrow \infty} \frac{1}{e^b} = 0. \text{ロピタルの定理より} \lim_{b \rightarrow \infty} b e^{-b} = \lim_{b \rightarrow \infty} \frac{b}{e^b} = \lim_{b \rightarrow \infty} \frac{1}{e^b} = 0. \text{よって与式} = 1.$$

$$(4) \text{与式} = \lim_{b \rightarrow \infty} \int_1^b \frac{1}{x(x^2+1)} dx = \lim_{b \rightarrow \infty} \int_1^b \left(\frac{1}{x} - \frac{x}{x^2+1} \right) dx = \lim_{b \rightarrow \infty} \left[\log|x| - \frac{1}{2} \log|x^2+1| \right]_1^b$$

$$= \lim_{b \rightarrow \infty} \left(\log b - \frac{1}{2} \log(b^2+1) - \log 1 + \frac{1}{2} \log 2 \right) = \lim_{b \rightarrow \infty} (\log b - \log \sqrt{b^2+1}) + \log \sqrt{2} = \lim_{b \rightarrow \infty} \log \frac{b}{\sqrt{b^2+1}} + \log \sqrt{2}$$

$$= \lim_{b \rightarrow \infty} \log \frac{1}{\sqrt{1 + \frac{1}{b^2+1}}} + \log \sqrt{2} = \log 1 + \log \sqrt{2} = \log \sqrt{2}.$$

$$(5) \text{ 与式} = \lim_{\varepsilon \rightarrow +0, b \rightarrow \infty} \int_{\varepsilon}^b \frac{dx}{\sqrt[3]{x^2}} = \lim_{\varepsilon \rightarrow +0, b \rightarrow \infty} \int_{\varepsilon}^b x^{-\frac{2}{3}} dx = \lim_{\varepsilon \rightarrow +0, b \rightarrow \infty} \left[3x^{\frac{1}{3}} \right]_{\varepsilon}^b = \lim_{\varepsilon \rightarrow +0, b \rightarrow \infty} \left(3b^{\frac{1}{3}} - 3\varepsilon^{\frac{1}{3}} \right) = \infty.$$

よって存在しない。

$$(6) \text{ 与式} = \lim_{\varepsilon \rightarrow +0, \varepsilon' \rightarrow +0} \left(\int_{-1}^{-\varepsilon} \frac{dx}{x^4} + \int_{\varepsilon'}^1 \frac{dx}{x^4} \right) = \lim_{\varepsilon \rightarrow +0} \int_{-1}^{-\varepsilon} x^{-4} dx + \lim_{\varepsilon' \rightarrow +0} \int_{\varepsilon'}^1 x^{-4} dx$$

$$= \lim_{\varepsilon \rightarrow +0} \left[-\frac{1}{3} x^{-3} \right]_{-1}^{-\varepsilon} + \lim_{\varepsilon' \rightarrow +0} \left[-\frac{1}{3} x^{-3} \right]_{\varepsilon'}^1 = \lim_{\varepsilon \rightarrow +0} \left(-\frac{1}{3} (-\varepsilon)^{-3} + \frac{1}{3} (-1)^{-3} \right) + \lim_{\varepsilon' \rightarrow +0} \left(-\frac{1}{3} + \frac{1}{3} (\varepsilon')^{-3} \right).$$

$$= \lim_{\varepsilon \rightarrow +0} \frac{1}{3\varepsilon^3} - \frac{1}{3} - \frac{1}{3} + \lim_{\varepsilon' \rightarrow +0} \frac{1}{3\varepsilon'^3} = \infty. \text{ よって存在しない.}$$