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第9章 応用数学

§.1 複素数

$$1.1 \quad (1) \quad e^{\frac{\pi}{2}i} = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$(2) \quad \exp(2-i\frac{3}{2}i) = e^2 (\cos(-\frac{3}{2}i) + i \sin(-\frac{3}{2}i)) = e^{2i}$$

$$(3) \quad e^{-i\frac{\pi}{3}} = \cos(-\frac{\pi}{3}) + i \sin(-\frac{\pi}{3}) = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

$$(4) \quad e^{i\frac{\pi}{3}} = \cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

$$(5) \quad \sqrt{i} = (e^{\frac{\pi}{2}i})^{\frac{1}{2}} = e^{\frac{\pi}{4}i} = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \\ = (e^{\frac{5\pi}{2}i})^{\frac{1}{2}} = e^{\frac{5\pi}{4}i} = \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$$

$$(6) \quad (1 - \sqrt{3}i)^6 = \left\{ 2 \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i \right) \right\}^6 = (2e^{-\frac{\pi}{3}i})^6 \\ = 2^6 e^{-2\pi i} = 64 (\cos(-2\pi) + i \sin(-2\pi)) = 64$$

$$1.2 \quad (1) \quad i+1 = \sqrt{2} \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = \sqrt{2} (\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}) = \sqrt{2} e^{i\frac{\pi}{4}}$$

$$(2) \quad 1 + \sqrt{3}i = 2 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = 2 (\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}) = 2e^{i\frac{\pi}{3}}$$

$$(3) \quad -3 = 3(-1) = 3(\cos \pi + i \sin \pi) = 3e^{i\pi}$$

$$(4) \quad \sqrt{3} + i = 2 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = 2 (\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}) = 2e^{i\frac{\pi}{6}}$$

1.3 $x^2 - 5x^2 + ax + b = 0$ 1根 $1 + \sqrt{2}i$ 共役複素 $1 - \sqrt{2}i$ 是根

$$\therefore (x-1)^2 + 2 = x^2 - 2x + 3 = (x-d)^2 + 2 \quad \text{割り, 切り}$$

$$\therefore x^2 - 5x^2 + ax + b = (x-d)(x^2 - 2x + 3)$$

$$-5 = -d - 2 \quad a = 3 + 2d \quad b = -d \cdot 3$$

$$d = 3 \quad a = 9 \quad b = -9$$

$$1.4 \quad (\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

$$\cos 3\theta + i \sin 3\theta = (\cos \theta + i \sin \theta)^3 = \cos^3 \theta + 3i \cos^2 \theta \sin \theta - 3 \cos \theta \sin^2 \theta - i \sin^3 \theta \\ = \cos^3 \theta - 3 \cos \theta \sin^2 \theta + i (3 \cos^2 \theta \sin \theta - \sin^3 \theta)$$

$$\therefore \cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta = 3 \sin \theta - 4 \sin^3 \theta$$

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$$1.5 \quad z(s) = \frac{1}{s^2 + 2bs + 1}$$

$$(1) \quad z(i\omega) = \frac{1}{-\omega^2 + 2b i \omega + 1} = \frac{1}{1 - \omega^2 + i 2b \omega} = \frac{1 - \omega^2 - 2b i \omega}{(1 - \omega^2)^2 + 4b^2 \omega^2}$$

$$\therefore \text{実部 } \operatorname{Re}(z) = \frac{1 - \omega^2}{(1 - \omega^2)^2 + 4b^2 \omega^2} \quad \text{虚部 } \operatorname{Im}(z) = \frac{-2b \omega}{(1 - \omega^2)^2 + 4b^2 \omega^2}$$

$$|z| = \frac{1}{\sqrt{(1 - \omega^2)^2 + 4b^2 \omega^2}}$$

$$(2) \quad (1 - \omega^2)^2 + 4b^2 \omega^2 = \omega^4 + 2(2b^2 - 1)\omega^2 + 1$$

$$= (\omega^2 + 2b^2 - 1)^2 + 4b^2 - 4b^4$$

$$2b^2 - 1 \leq 0 \text{ のとき } |z| \text{ の最大値 } \frac{1}{2b \sqrt{1 - b^2}}$$

$$2b^2 - 1 > 0 \text{ のとき } |z| \text{ の最大値 } 1$$

$$1.6 (1) \quad |z - a| + |z + a| = 2|b| \quad \text{必要十分条件}$$

$$2|b| = |z - a| + |z + a| \geq |z - a - z - a| = 2|a|$$

$$|b| \geq |a|$$

$$(2) \quad a = r e^{i\theta} \text{ と表せば } |a| = r, \quad z_1 = z e^{-i\theta} \text{ と } r < r \quad a e^{-i\theta} = r$$

$$|z - a| = |z_1 - r|, \quad |z + a| = |z_1 + r|$$

$$z_1 = x + i y \text{ とする}$$

$$|z_1 - r| + |z_1 + r| = \sqrt{(x-r)^2 + y^2} + \sqrt{(x+r)^2 + y^2} = 2|b|$$

$$\frac{1}{\sqrt{(x-r)^2 + y^2} + \sqrt{(x+r)^2 + y^2}} = \frac{1}{2|b|}$$

$$\sqrt{(x+r)^2 + y^2} - \sqrt{(x-r)^2 + y^2} = \frac{2xr}{|b|^2}$$

$$\sqrt{(x+r)^2 + y^2} = |b| + \frac{xr}{|b|^2}$$

$$\therefore (x+r)^2 + y^2 = |b|^2 + 2xr + \frac{x^2 r^2}{|b|^2}$$

$$\left(1 - \frac{r^2}{|b|^2}\right) x^2 + y^2 = |b|^2 - r^2 \quad (r^2 = a^2)$$

$$\frac{x^2}{|b|^2} + \frac{y^2}{|b|^2 - r^2} = 1 \quad \therefore -a, a \text{ を中心とする楕円}$$

$|z_1|$ は右円の中心から右円周上の点までの距離

$$\therefore |z| = |z_1| \text{ の最大値 } |b|, \text{ 最小値 } \sqrt{|b|^2 - |a|^2}$$

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1.7 Pz 表示複素数 $z = re^{i\theta}$ と $z^k = r^k e^{ik\theta}$ $i = e^{i\frac{\pi}{2}}$
 $\therefore re^{i\theta} (e^{i\frac{\pi}{2}})^2 = r e^{i(\theta + \frac{\pi}{2})} = r(\cos(\theta + \frac{\pi}{2}) + i\sin(\theta + \frac{\pi}{2}))$
 270° 回転云

1.8 $z\bar{z} + 2z = 2i$ $z = x + iy$ $x, y < 0$
 $x^2 + y^2 + 2x + 2iy = 2i$
 $\therefore x^2 + y^2 + 2x = 0$ $y = 1$ $\therefore x^2 + 2x + 1 = 0$ $x = -1$
 $\therefore z = -1 + i$

1.9 $x^4 - 81i = 0$ $x = r e^{i\theta}$ $81i = 3^4 e^{i(2k + \frac{1}{2})\pi}$
 $\therefore x^4 = 81i$ $\therefore r^4 e^{i4\theta} = 3^4 e^{i(2k + \frac{1}{2})\pi}$
 $\therefore r = 3$ $4\theta = (2k + \frac{1}{2})\pi$ $\theta = \frac{k}{2}\pi + \frac{1}{8}\pi$ $k = 0, 1, 2, 3$
 $\cos \frac{2\pi}{8} = \frac{1}{2}(1 + \cos \frac{\pi}{4}) = \frac{1}{4}(2 + \sqrt{2})$ $\cos \frac{\pi}{8} = \frac{1}{2}\sqrt{2 + \sqrt{2}}$
 $\sin \frac{2\pi}{8} = \frac{1}{2}(1 - \cos \frac{\pi}{4}) = \frac{1}{4}(2 - \sqrt{2})$ $\sin \frac{\pi}{8} = \frac{1}{2}\sqrt{2 - \sqrt{2}}$
 $\therefore X = \pm \frac{3}{2}(\sqrt{2 + \sqrt{2}} + i\sqrt{2 - \sqrt{2}}), \pm \frac{3}{2}(-\sqrt{2 - \sqrt{2}} + i\sqrt{2 + \sqrt{2}})$

1.10 $w = \frac{1}{z}$ $x^2 + y^2 = 4$ $z = x + iy$ $w = u + iv$
 $|w| = \frac{1}{|z|} = \frac{1}{2}$ $\therefore |w|^2 = \frac{1}{4}$
 $\therefore u^2 + v^2 = \frac{1}{4}$ 原点を中心とした半径 $\frac{1}{2}$ の円に写像される

§. 2 複素関数

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2.1

$$w = \frac{z-1}{z+i} \quad |z|=2$$

$$w(z+i) = z-1 \quad (w-1)z = -1(w+1) \quad z = \frac{-1(w+1)}{w-1}$$

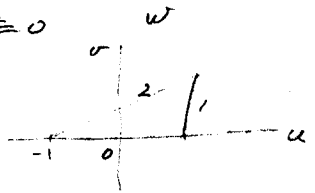
$$\left| \frac{w+1}{w-1} \right| = 2 \quad \therefore |w+1| = 2|w-1|$$

$$w = u+iv \quad u, v \in \mathbb{R} \quad (u+1)^2 + v^2 = 4\{(u-1)^2 + v^2\}$$

$$4(u-1)^2 - (u+1)^2 + 3v^2 = 0 \quad 3u^2 - 10u + 3 + 3v^2 = 0$$

$$3\left(u - \frac{5}{3}\right)^2 + 3v^2 = \frac{25}{3} - 3 = \frac{16}{3}$$

$$\therefore \left(u - \frac{5}{3}\right)^2 + v^2 = \frac{16}{9} \quad \text{円 (中心 } \frac{5}{3}, \text{ 半径 } \frac{4}{3})$$



2.2

$$w = \cosh z \quad z \text{ 平面上 } x > 0, \quad 0 \leq y \leq \pi$$

$$z = x+iy \quad w = u+iv \quad u, v \in \mathbb{R}$$

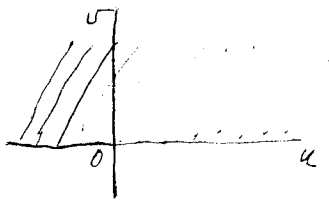
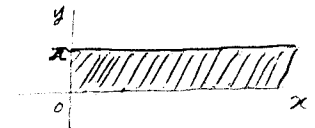
$$\cosh z = \frac{1}{2}(e^{x+iy} + e^{-x-iy})$$

$$= \frac{1}{2}\{e^x(\cos y + i \sin y) + e^{-x}(\cos y - i \sin y)\}$$

$$= \frac{1}{2}(e^x + e^{-x})\cos y + \frac{i}{2}(e^x - e^{-x})\sin y$$

$$u = \frac{1}{2}(e^x + e^{-x})\cos y, \quad v = \frac{1}{2}(e^x - e^{-x})\sin y$$

$$0 \leq y \leq \pi \quad v \geq 0$$



2.3

$$z = x+iy, \quad w = u+iv \quad w = z^2$$

$$u+iv = x^2 - y^2 + 2xyi$$

$$\begin{cases} u = x^2 - y^2 & x = a \text{ かつ } y = b & u = a^2 - b^2 & a \neq 0 \text{ かつ } b \neq 0 \\ v = 2xy & & v = 2ab & u = a^2 - \frac{v^2}{4a^2} \end{cases}$$

$$a = 0 \text{ かつ } b \neq 0$$

$$u \leq 0, \quad v = 0$$

$$y = b \text{ かつ } x \neq 0$$

$$b \neq 0 \text{ かつ } x = 0$$

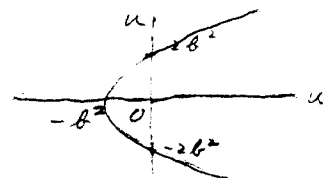
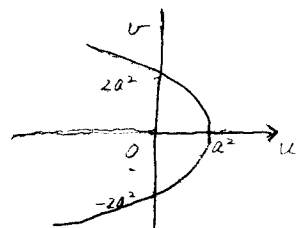
$$b = 0 \text{ かつ } x \neq 0$$

$$\begin{cases} u = x^2 - b^2 \\ v = 2bx \end{cases}$$

$$u = \frac{v^2}{4b^2} - b^2$$

$$u \geq 0$$

$$v = 0$$



i) $v=0 \quad xy=0$

ii) $v = 4\sqrt{4-u} \quad v^2 = 16(4-u) \quad x^2y^2 = 4(4-x^2+y^2)$

$$x^2y^2 + 4x^2 - 4y^2 = 16 \quad x^2(y^2+4) - 4(y^2+4) = 0$$

$$(x^2-4)(y^2+4) = 0 \quad x = 2 \quad (v \geq 0)$$

iii) $v = 2\sqrt{1+u}$ $v^2 = 4(1+u)$ $x^2y^2 = (1+x^2-y^2)$

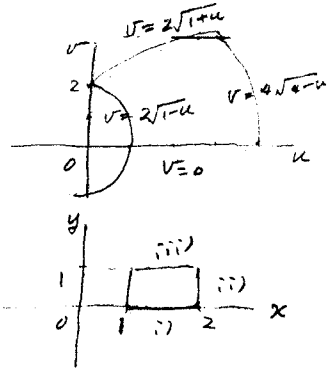
$x^2y^2 = x^2 + y^2 - 1 = 0$

$(x^2+1)(y^2-1) = 0$ $y = 1, (v \geq 0)$

iv) $v = 2\sqrt{1-u}$ $v^2 = 4(1-u)$ $x^2y^2 = 1-x^2-y^2$

$x^2y^2 + x^2 + y^2 - 1 = 0$ $(x^2-1)(y^2+1) = 0$

$x = 1$



2. $e^{ix} = \cos x + i \sin x$

(1) $e^{ix} = \cos x + i \sin x$ $\cos x = \frac{1}{2}(e^{ix} + e^{-ix})$

$e^{-ix} = \cos x - i \sin x$ $\sin x = \frac{1}{2i}(e^{ix} - e^{-ix})$

$\therefore \tan x = \frac{e^{ix} - e^{-ix}}{i(e^{ix} + e^{-ix})}$

(2) $\sin x = \frac{1}{2i}(e^{ix} - e^{-ix})$

$\sin x = \frac{1}{2i}(e^{ix} + e^{-ix})$

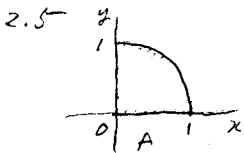
$\sin^{(n)} x = \frac{(i)^n}{2i}(e^{ix} - (-1)^n e^{-ix})$

$\sin x = 0$ $\sin' x = 1$ $n = 2m$ $\sin^{(2m)} x = 0$

$n = 2m+1$ $\sin^{(2m+1)} x = \frac{(-1)^m}{2} \cdot 2 = (-1)^m$

$\therefore \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^m \frac{x^{2m+1}}{(2m+1)!} + \dots$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^m \frac{x^{2m}}{(2m)!} + \dots$



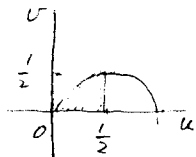
$z = x + iy$ $x^2 + y^2 \leq 1$ $x \geq 0$ $y \geq 0$

$w = \frac{z}{z+1}$ $w(z+1) = z$ $z(w-1) = -w$

$z = \frac{w}{1-w} = \frac{u+iv}{1-u-iv} = \frac{u(1-u)-v^2 + i(uv+u-v-u^2)}{(1-u)^2+v^2}$

$x = \frac{u(1-u)-v^2}{(1-u)^2+v^2}$ $y = \frac{v}{(1-u)^2+v^2}$

$v \geq 0$ $u-u^2-v^2 \geq 0$ $\frac{1}{4} \geq (u-\frac{1}{2})^2 + v^2$



$|\frac{w}{w-1}| \leq 1$ $|w| \leq |w-1|$ $u \leq \frac{1}{2}$

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2.6 $w = f(z) = \sin z$

$z = x + iy$

(1) $\sin z = \frac{1}{2i} (e^{iz} - e^{-iz}) = \frac{1}{2i} (e^{i(x+iy)} - e^{-i(x+iy)})$

$$= \frac{1}{2i} \{ e^{-y} (\cos x + i \sin x) - (\cos x - i \sin x) e^y \}$$

$$= \frac{1}{2i} (e^{-y} - e^y) \cos x + \frac{1}{2} (e^{-y} + e^y) \sin x$$

$$= \frac{i}{2} (e^y - e^{-y}) \cos x + \frac{1}{2} (e^y + e^{-y}) \sin x$$

$$= \sin x \cosh y + i \cos x \sinh y$$

(2) $u + iv = \sin x \cosh y + i \cos x \sinh y$

$$\therefore u = \sin x \cosh y$$

$$v = \cos x \sinh y$$

$$y = 0 \quad v = 0 \quad 0 \leq u \leq \frac{1}{\sqrt{2}}$$

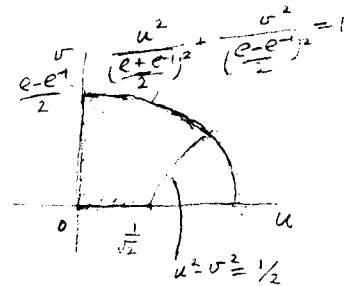
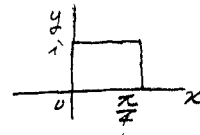
$$x = \frac{\pi}{4} \quad u = \frac{1}{\sqrt{2}} \cosh y \quad v = \frac{1}{\sqrt{2}} \sinh y$$

$$u^2 - v^2 = \frac{1}{2}$$

$$y = 1 \quad u = \frac{e + e^{-1}}{2} \sin x \quad v = \frac{e - e^{-1}}{2} \cos x$$

$$\left(\frac{e + e^{-1}}{2}\right)^2 - \left(\frac{e - e^{-1}}{2}\right)^2 = 1$$

$$x = 0 \quad u = 0 \quad 0 \leq v \leq \frac{e - e^{-1}}{2}$$



2.7 $w = \frac{i(1-z)}{1+z} \quad |z| < 1$

$$w(1+z) = i - iz$$

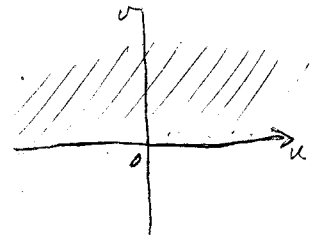
$$z(u+i) = i - iw$$

$$z = \frac{i - iw}{i + u} \quad \left| \frac{i - iw}{i + u} \right| < 1$$

$$\therefore |i - w| < |i + w|$$

$$w = u + iv \quad \therefore |i - w|^2 < |i + w|^2$$

$$v > 0$$



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§ 3 正則関数

$$3.1 \quad u = \sin x \cosh y \quad u_x = v_y \quad u_y = -v_x \quad v(0,0) = 0$$

$$v_y = u_x = \cos x \cosh y$$

$$v_x = -u_y = -\sin x \sinh y$$

$$\therefore v = \cos x \sinh y + C \quad v(0,0) = 0 \quad \therefore C = 0$$

$$\therefore v(x,y) = \cos x \sinh y$$

$$3.2 \quad f(z) = u(x,y) + i v(x,y) \quad z = x + iy \quad \text{と } \bar{z} = x - iy$$

$$f(z) \text{ は正則} \quad \therefore u_x = v_y \quad u_y = -v_x$$

$$u = v \text{ のとき} \quad u_x = u_y \quad u_y = -u_x \quad \therefore u_x = u_y = 0$$

$\therefore u, v$ は定数

$$3.3 \quad u = -6x^2y + 2y^3 \text{ のとき } v(x,y) \text{ を定め } w = u + iv \text{ が正則$$

$$\therefore u_x = v_y \quad u_y = -v_x \quad \therefore v_y = -12xy \quad v_x = -(-6x^2 + 6y^2) = 6x^2 - 6y^2$$

$$\therefore v = -6xy^2 + \varphi_1(x) \quad v = 2x^3 - 6xy^2 + \varphi_2(y) \quad \varphi_1 \text{ は } x \text{ の関数}$$

$$\therefore v = 2x^3 - 6xy^2$$

φ_2 は y の定数

$$3.4 \quad w = (x^2 + axy + by^2) + i(cx^2 + dx^2 + y^2)$$

$$u_x = v_y \quad \therefore 2x + ay = dx + 2y \quad d = a = 2$$

$$u_y = -v_x \quad ax + 2by = -2cx - dy \quad c = -1 \quad b = -1$$

$$\therefore w = x^2 + 2xy - y^2 + i(-x^2 + 2x^2 + y^2)$$

$$= x^2 + 2xyi - y^2 - i(x^2 + 2xyi - y^2) = z^2 - i z^2$$

$$\therefore f(z) = z^2 - i z^2$$

$$3.5 \quad f(z) = u + iv \quad z = x + iy \quad u_x = v_y \quad u_y = -v_x$$

$$u + v = x^3 + 3x^2y - 3xy^2 - y^3 - x^2 + 2xy + y^2 + 1$$

$$u_x + v_x = u_x - u_y = 3x^2 + 6xy - 3y^2 - 2x + 2y \quad 2u_x = 6x^2 - 6y^2 + 4y$$

$$u_y + v_y = u_y + u_x = 3x^2 - 6xy - 3y^2 + 2x + 2y \quad 2u_y = -12xy + 4x$$

$$u = x^3 - 3y^2x + 2xy$$

$$v = 3x^2y - y^3 - x^2 + y^2 + 1$$

$$f(z) = z^3 - i z^2 + i$$

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§. 4. 複素関数の積分

ローラン展開 $f(z) = \sum_{-\infty}^{\infty} b_n (z-a)^n$ $b_n = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z-a)^{n+1}} dz$

4.1 (1) $f(z) = \frac{1}{z^2(z-i)z-2i} = \frac{1}{(z-2)(z+i)}$ 極 $2, -i$

i) $z=2e^{i\theta}$

$$b_n = \frac{1}{2\pi i} \int_C \frac{1}{(z-2)^{n+2}(z+i)} dz$$

$$= \frac{1}{2\pi i} \int_0^{2\pi} \frac{i e^{i\theta}}{(2+i+e^{i\theta}) e^{i(n+2)\theta}} d\theta$$

$$= \frac{1}{2\pi i} \int_0^{2\pi} \frac{i e^{-i(n+2)\theta}}{(2+i+e^{i\theta})} d\theta$$

$$\frac{1}{2+i+e^{i\theta}} = \frac{1}{2+i} \frac{1}{1 + \frac{e^{i\theta}}{2+i}} = \frac{1}{2+i} \left(1 - \frac{e^{i\theta}}{2+i} + \left(\frac{e^{i\theta}}{2+i}\right)^2 - \dots + \left(\frac{-e^{i\theta}}{2+i}\right)^k + \dots \right)$$

$$\left| \frac{e^{i\theta}}{2+i} \right| = \frac{1}{|2+i|} < 1 \quad \text{収束}$$

$$\int_0^{2\pi} e^{-i m \theta} d\theta = 0 \quad (m \neq 0) \quad \int_0^{2\pi} e^{0} d\theta = 2\pi$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{2+i} \left(\frac{-1}{2+i}\right)^{n+1} d\theta = \frac{(-1)^{n+1}}{(2+i)^{n+2}} \quad n \geq -1$$

$$\therefore b_n = \frac{(-1)^{n+1}}{(2+i)^{n+2}}$$

$$f(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2+i)^{n+2}} (z-2)^{n-1}$$

ii) $z=-i z^{-1}$

$$b_n = \frac{1}{2\pi i} \int_C \frac{1}{(z-2)(z+i)^{n+2}} dz$$

$$= \frac{1}{2\pi i} \int_0^{2\pi} \frac{i e^{i\theta} e^{-i(n+2)\theta}}{-(2+i-e^{i\theta})} d\theta$$

$$= \frac{-i}{2\pi} \int_0^{2\pi} \frac{e^{-i(n+1)\theta}}{(2+i)\left(1 - \frac{e^{i\theta}}{2+i}\right)} d\theta$$

$$= \frac{-i}{2\pi} \frac{1}{2+i} \left(\frac{1}{2+i}\right)^{n+1} \cdot 2\pi = -\frac{1}{(2+i)^{n+2}} \quad n \geq -1$$

$$\therefore b_n = -\frac{1}{(2+i)^{n+2}}$$

$$\therefore f(z) = -\sum_{n=0}^{\infty} \frac{1}{(2+i)^{n+2}} (2+i)^{n-1}$$

C: $|z+i|=1$

$z = -i + e^{i\theta}$

$dz = i e^{i\theta} d\theta$

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$$\begin{aligned}
 (2) \quad f(z) &= \frac{z}{(z+1)(z-2)} \quad 1 < |z| < 2 \\
 &= \frac{1}{3} \left(\frac{z}{z-2} + \frac{1}{z+1} \right) \\
 &= \frac{1}{3} \left\{ \frac{-1}{1-\frac{z}{2}} + \frac{1}{1+z} \right\} = \frac{1}{3} \left\{ \sum_{n=0}^{\infty} (-1) \left(\frac{z}{2}\right)^n + \sum_{n=0}^{\infty} (-z)^n \right\} \\
 &= \frac{1}{3} \sum_{n=0}^{\infty} \left\{ (-1)^n - \frac{1}{2^n} \right\} z^n
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad f(z) &= \frac{1}{z(z-1)^3} \quad 0 < |z-1| < 1 \\
 &= \frac{1}{u^3(u+1)} \quad \begin{array}{l} z-1=u \quad 0 < u < 1 \\ z=u+1 \end{array} \\
 &= \frac{1}{u^3} \sum_{n=0}^{\infty} (-u)^n = \sum_{n=0}^{\infty} (-1)^n u^{n-3} = \sum_{n=0}^{\infty} (-1)^n (z-1)^{n-3}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad f(z) &= \frac{1}{z(z-1)^3} \quad |z| < 1 \\
 \frac{1}{1-z} &= \sum_{n=0}^{\infty} z^n = u \quad \frac{1}{(1-z)^2} = \sum_{n=1}^{\infty} n z^{n-1} \quad \frac{1}{(1-z)^3} = \sum_{n=2}^{\infty} \frac{n(n-1)}{2} z^{n-2} \\
 \frac{1}{(1-z)^3} &= \sum_{n=0}^{\infty} \frac{(n+2)(n+1)}{2} z^n \\
 \therefore f(z) &= - \sum_{n=0}^{\infty} \frac{(n+2)(n+1)}{2} z^{n-1}
 \end{aligned}$$

$$\begin{aligned}
 4.2 \quad \int_C \frac{1}{(z+i)^2(z-i)} dz \quad C: |z+i|=1 \\
 &= \int_C \frac{1}{(z+i)^2(z-i)} dz = \int_C \frac{1}{(z+i)^2(z-i-2i)} dz \\
 &= \frac{1}{-2i} \int_C \frac{1}{(z+i)^2} \frac{1}{1-\frac{z+i}{2i}} dz = \frac{-1}{2i} \int_C \sum_{n=0}^{\infty} \frac{(z+i)^{n-2}}{(2i)^n} dz \quad \begin{array}{l} z+i = e^{i\theta} \\ dz = i e^{i\theta} d\theta \end{array} \\
 &= -\frac{1}{2} \int_0^{2\pi} \sum_{n=0}^{\infty} \frac{1}{(2i)^n} e^{i(n-1)\theta} d\theta = -\frac{1}{2} \cdot \frac{1}{2i} \cdot 2\pi = \frac{-\pi}{2i} = \frac{\pi}{2}i
 \end{aligned}$$

$$\begin{aligned}
 4.3 \quad (1) \quad \int_0^{\infty} \frac{2x^2-1}{x^4+5x^2+4} dx &= \int_0^{\infty} \left(\frac{3}{x^2+4} - \frac{1}{x^2+1} \right) dx \\
 &= \left[\frac{3}{2} \tan^{-1} \frac{x}{2} - \tan^{-1} x \right]_0^{\infty} = \frac{3}{2} \cdot \frac{\pi}{2} - \frac{\pi}{2} = \frac{\pi}{4}
 \end{aligned}$$

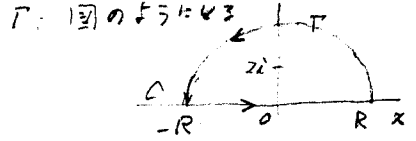
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$$f(z) = \sum_{n=0}^{\infty} b_n (z-a)^n \quad b_n = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z-a)^{n+1}} dz$$

4.3 (2) $\int_{-\infty}^{\infty} \frac{\cos \pi x}{x^2+2x+5} dx = \int_{-\infty}^{\infty} \frac{\cos \pi x}{(x+1)^2+4} dx$ $x+1=z$ とおくと $x=z-1$ $\cos \pi x = -\cos \pi z$

$$\int_{-\infty}^{\infty} \frac{-\cos \pi z}{z^2+4} dz \quad \int_{\Gamma} \frac{\cos \pi z}{z^2+4} dz$$



$$\frac{1}{2\pi i} \int_C \frac{-e^{i\pi z}}{z^2+4} dz = \text{Res}(zi)$$

$$\text{Res}(zi) = \lim_{z \rightarrow zi} (z-zi) \frac{e^{i\pi z}}{z^2+4} = \lim_{z \rightarrow zi} \frac{e^{i\pi z}}{z+2i} = \frac{e^{-2\pi}}{4i}$$

$$\therefore \int_C \frac{e^{i\pi z}}{z^2+4} dz = \frac{\pi e^{-2\pi}}{2}$$

$$\int_C \frac{e^{i\pi z}}{z^2+4} dz = \int_{-R}^0 \frac{e^{i\pi z}}{z^2+4} dz + \int_0^R \frac{e^{i\pi z}}{z^2+4} dz + \int_{\Gamma} \frac{e^{i\pi z}}{z^2+4} dz$$

$$\Gamma: z = Re^{i\theta} = R(\cos \theta + i \sin \theta) \quad 0 \leq \theta \leq \pi$$

$$\left| \int_{\Gamma} \frac{e^{i\pi z}}{z^2+4} dz \right| = \left| \int_0^{\pi} \frac{e^{i\pi R \cos \theta} e^{-\pi R \sin \theta}}{R^2 e^{i2\theta} + 4} i R e^{i\theta} d\theta \right| \leq \int_0^{\pi} \frac{R}{R^2-4} e^{-\pi R \sin \theta} d\theta \leq \frac{\pi R}{R^2-4}$$

$$\leq \frac{R}{R^2-4} \int_0^{\pi} e^{-\pi R \sin \theta} d\theta \leq \frac{\pi R}{R^2-4}$$

$$R \rightarrow \infty \text{ とき } \int_{\Gamma} \frac{e^{i\pi z}}{z^2+4} dz \rightarrow 0$$

$$\int_{-R}^0 \frac{e^{i\pi z}}{z^2+4} dz + \int_0^R \frac{e^{i\pi z}}{z^2+4} dz = \int_0^R \frac{e^{i\pi z} + e^{-i\pi z}}{z^2+4} dz$$

$$= \int_0^R \frac{2 \cos \pi z}{z^2+4} dz$$

$$2 \int_0^{\infty} \frac{\cos \pi z}{z^2+4} dz = \frac{\pi e^{-2\pi}}{2}$$

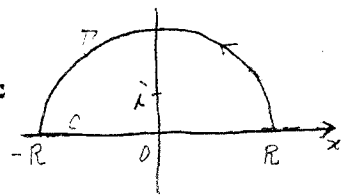
$$\int_{-\infty}^{\infty} \frac{\cos \pi x}{x^2+4} dx = \frac{\pi e^{-2\pi}}{2}$$

$$\therefore \int_{-\infty}^{\infty} \frac{\cos \pi x}{(x+1)^2+4} dx = \int_{-\infty}^{\infty} \frac{-\cos \pi z}{z^2+4} dz = -\frac{\pi e^{-2\pi}}{2}$$

4.4 (1) $f(z) = \frac{ze^{iz}}{1+z^2}$ $\text{Res}(f; i) = \lim_{z \rightarrow i} (z-i) \frac{ze^{iz}}{z^2+1} = \lim_{z \rightarrow i} \frac{ze^{iz}}{z+i}$
 $= \frac{i e^{-1}}{2i} = \frac{e^{-1}}{2}$

(2) $\frac{1}{2\pi i} \int_C \frac{ze^{iz}}{1+z^2} dz = \frac{e^{-1}}{2}$

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$$\therefore \int_C \frac{ze^{iz}}{1+z^2} dz = \pi e^{-1} i$$

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$$\int_C \frac{ze^{iz}}{1+z^2} dz = \int_{-R}^0 \frac{xe^{-ix}}{1+x^2} dx + \int_0^R \frac{xe^{ix}}{1+x^2} dx + \int_{\Gamma} \frac{ze^{iz}}{1+z^2} dz \quad \Gamma: z = Re^{i\theta}$$

$$\left| \int_{\Gamma} \frac{ze^{iz}}{1+z^2} dz \right| = \left| \int_0^{\pi} \frac{Re^{i\theta} e^{iR\cos\theta} e^{-R\sin\theta}}{1+R^2e^{i2\theta}} iRe^{i\theta} d\theta \right|$$

$$= \left| \int_0^{\pi} \frac{iR^2 e^{iR\cos\theta} e^{-R\sin\theta}}{1+R^2e^{i2\theta}} d\theta \right| \leq \frac{R^2}{R^2-1} \int_0^{\pi} e^{-R\sin\theta} d\theta$$

$$\int_0^{\pi} e^{-R\sin\theta} d\theta = 2 \int_0^{\frac{\pi}{2}} e^{-R\sin\theta} d\theta \leq 2 \left\{ \int_0^{\epsilon} d\theta + \int_{\epsilon}^{\frac{\pi}{2}} e^{-R\sin\theta} d\theta \right\}$$

$$\leq 2(\epsilon + e^{-R\sin\epsilon} (\frac{\pi}{2} - \epsilon)) \quad 0 < \epsilon < 0 < \frac{\pi}{2}$$

$$0 < \sin\epsilon < \sin\theta < 1$$

$$0 > -R\sin\epsilon > -R\sin\theta > -1$$

$$\Rightarrow z^n R \rightarrow \infty \text{ as } R \rightarrow \infty \Rightarrow e^{-R\sin\epsilon} \rightarrow 0$$

$$\Rightarrow \int_0^{\pi} e^{-R\sin\theta} d\theta \rightarrow 0 \quad \therefore \left| \int_{\Gamma} \frac{ze^{iz}}{1+z^2} dz \right| \rightarrow 0$$

$$\therefore \int_{-\infty}^0 \frac{xe^{ix}}{1+x^2} dx + \int_0^{\infty} \frac{xe^{ix}}{1+x^2} dx = \pi e^{-1}$$

$$\therefore \int_0^{\infty} \frac{x(e^{ix} - e^{-ix})}{1+x^2} dx = \pi e^{-1} \quad \therefore \int_0^{\infty} \frac{x \sin x}{1+x^2} dx = \frac{\pi}{2e}$$

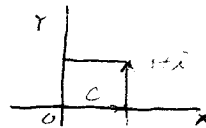
$$\therefore \int_{-\infty}^{\infty} \frac{x \sin x}{1+x^2} dx = \pi e^{-1}$$

4.5 $f(z) = z^2$

$$\int_C f(z) dz = \int_0^1 x^2 dx + \int_1^{1+i} (1+iy)^2 i dy$$

$$= \left[\frac{x^3}{3} \right]_0^1 + i \left[y + iy^2 - \frac{y^3}{3} \right]_0^1$$

$$= \frac{1}{3} + i \left(\frac{2}{3} + i \right) = \frac{-2}{3} + \frac{2}{3}i$$



4.6 (1) $z^4 = -1 \quad z = e^{i\theta} \text{ with } 0 < \theta < 2\pi$

$$e^{i4\theta} = e^{i\pi(2k+1)} \quad \therefore 4\theta = (2k+1)\pi \quad \theta = \frac{k\pi}{2} + \frac{\pi}{4}, \quad k = 0, 1, 2, 3$$

$$z = \omega \left(\frac{k\pi}{2} + \frac{\pi}{4} \right) + i \omega \left(\frac{k\pi}{2} + \frac{\pi}{4} \right)$$

$$k=0, \quad z = \frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}}, \quad k=1, \quad z = -\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}}$$

$$k=2, \quad z = -\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}}, \quad k=3, \quad z = \frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}}$$

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$$4.6 (z) \oint_C \frac{1}{z^2+1} dz$$

$$0 < R < 1 \text{ and } \infty$$

$$\oint_C \frac{1}{z^2+1} dz = 0$$

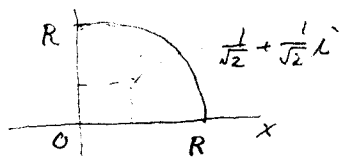
$$1 < R \text{ and } \infty$$

$$\oint_C \frac{1}{z^2+1} dz = 2\pi i \operatorname{Res}(f, \frac{1}{\sqrt{2}}(1+i))$$

$$\operatorname{Res}(f, \frac{1}{\sqrt{2}}(1+i)) = \lim_{z \rightarrow \frac{1+i}{\sqrt{2}}} (z - \frac{1+i}{\sqrt{2}}) \frac{1}{(z - \frac{1+i}{\sqrt{2}})(z + \frac{1+i}{\sqrt{2}})(z^2+1)}$$

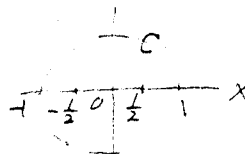
$$= \frac{1}{\sqrt{2}(1+i)2i} = \frac{-i(1-i)}{4\sqrt{2}} = \frac{-1-i}{4\sqrt{2}}$$

$$\therefore \oint_C \frac{1}{z^2+1} dz = \frac{\pi i(-1-i)}{2\sqrt{2}} = \frac{\pi(1-i)}{2\sqrt{2}} = \frac{\pi(\sqrt{2}-i\sqrt{2})}{4}$$



$$4.7 \quad C: |z|=1$$

$$\int_C \frac{1}{(z^2+1)(z^2-1)^2} dz$$



$$\therefore \int_C \frac{1}{(z^2+1)(z^2-1)^2} dz = \operatorname{Res}(\frac{1}{2}) + \operatorname{Res}(-\frac{1}{2})$$

$$\operatorname{Res}(-\frac{1}{2}) = \lim_{z \rightarrow -\frac{1}{2}} (z + \frac{1}{2}) \frac{1}{(z^2+1)(z^2-1)^2} = -\frac{1}{8}$$

$$\operatorname{Res}(\frac{1}{2}) = \frac{1}{2} \lim_{z \rightarrow \frac{1}{2}} \frac{d}{dz} \frac{1}{(z^2+1)} = \frac{1}{2} \lim_{z \rightarrow \frac{1}{2}} \frac{-2(-z)}{(z^2+1)^2} = \frac{1}{2}$$

$$\therefore \int_C \frac{1}{(z^2+1)(z^2-1)^2} dz = 2\pi i \left(-\frac{1}{8} + \frac{1}{2}\right) = \frac{3}{4}\pi i$$

$$4.8 \quad f(z) = \frac{z-2}{z^2(z-1)^3} \quad \text{pole } 0, 2 \text{ 位, pole } 1, 3 \text{ 位,}$$

$$\operatorname{Res}(f, 0) = \lim_{z \rightarrow 0} \frac{d}{dz} \frac{z-2}{(z-1)^3} = \lim_{z \rightarrow 0} \frac{z-1-3(z-2)}{(z-1)^4} = \lim_{z \rightarrow 0} \frac{-2z+5}{(z-1)^4} = 5$$

$$\operatorname{Res}(f, 1) = \lim_{z \rightarrow 1} \frac{1}{2} \frac{d^2}{dz^2} \frac{z-2}{z^2} = \frac{1}{2} \lim_{z \rightarrow 1} \left(\frac{2}{z^3} - \frac{12}{z^4} \right) = -5$$

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$$4.9 \quad f(z) = e^z \quad z = x + iy \quad x, y \in \mathbb{R}$$

$$(*) \quad f(z) = e^{x+iy} = e^x (\cos y + i \sin y)$$

$$\therefore \operatorname{Re}(e^z) = e^x \cos y, \quad \operatorname{Im}(e^z) = e^x \sin y$$

$$(a) \quad u = e^x \cos y, \quad v = e^x \sin y$$

$$u_x = e^x \cos y, \quad v_y = e^x \cos y \quad \therefore u_x = v_y$$

$$u_y = -e^x \sin y, \quad v_x = e^x \sin y \quad \therefore u_y = -v_x$$

$$(b) \quad \int_C f(z) dz = \int_C e^z dz \quad C: |z|=1 \quad z = e^{i\theta}$$

$f(z)$ は C の内部で正則である

$$\therefore \int_C f(z) dz = 0$$

$$4.10 \quad z = t^2 + i t^3 \quad 0 \leq t \leq 1, \quad i = \sqrt{-1}$$

$$dz = (2t + i 3t^2) dt$$

$$\begin{aligned} \int_C e^z dz &= \int_0^1 e^{t^2 + i t^3} (2t + i 3t^2) dt \\ &= \int_0^1 e^{t^2} (\cos(t^3) + i \sin(t^3)) (2t + i 3t^2) dt \\ &= \int_0^1 e^{t^2} \{ 2t \cos(t^3) - 3t^2 \sin(t^3) \} dt + i \int_0^1 e^{t^2} \{ 3t^2 \cos(t^3) + 2t \sin(t^3) \} dt \\ &= [e^{t^2} \cos(t^3)]_0^1 + \int_0^1 e^{t^2} 3t^2 \sin(t^3) dt - \int_0^1 e^{t^2} 3t^2 \sin(t^3) dt \\ &\quad + i \{ [e^{t^2} \sin(t^3)]_0^1 - \int_0^1 e^{t^2} 3t^2 \cos(t^3) dt + \int_0^1 e^{t^2} 3t^2 \cos(t^3) dt \} \\ &= e \cos 1 - 1 + i e \sin 1 \end{aligned}$$

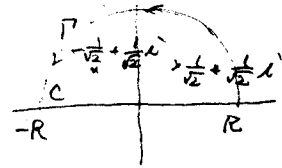
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$$4.11 \int_{-\infty}^{\infty} \frac{1}{x^2+1} dx$$

$$z^2+1=0$$

$$(z^2-1)(z^2+1)=0$$

$$z = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}, \quad -\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}, \quad \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}, \quad \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$$



$$\frac{1}{2\pi i} \int_C \frac{dz}{z^2+1} = \text{Res}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) + \text{Res}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$$

$$\text{Res}\left(\frac{1+i}{\sqrt{2}}\right) = \lim_{z \rightarrow \frac{1+i}{\sqrt{2}}} \frac{1}{(z + \frac{1+i}{\sqrt{2}})(z^2+1)} = \frac{1}{\sqrt{2}(1+i)2i} = \frac{-1-i}{4\sqrt{2}} = \frac{-1-i}{4\sqrt{2}}$$

$$\text{Res}\left(\frac{-1+i}{\sqrt{2}}\right) = \lim_{z \rightarrow \frac{-1+i}{\sqrt{2}}} \frac{1}{(z^2-1)(z - \frac{-1+i}{\sqrt{2}})} = \frac{1}{-2i\sqrt{2}(-1+i)} = \frac{1(-1-i)}{4\sqrt{2}} = \frac{-1-i}{4\sqrt{2}}$$

$$\therefore \int_C \frac{1}{z^2+1} dz = 2\pi i \left(\frac{-1-i}{4\sqrt{2}} + \frac{-1-i}{4\sqrt{2}} \right) = \frac{\pi}{\sqrt{2}}$$

$$\int_C \frac{dz}{z^2+1} = \int_{-R}^R \frac{1}{x^2+1} dx + \int_0^R \frac{1}{z^2+1} dz + \int_{\Gamma} \frac{1}{z^2+1} dz \quad \Gamma: |z|=R$$

$$= \int_{-R}^R \frac{1}{x^2+1} dx + \int_{\Gamma} \frac{1}{z^2+1} dz \quad z = Re^{i\theta}$$

$$\left| \int_{\Gamma} \frac{1}{z^2+1} dz \right| = \left| \int_0^{\pi} \frac{Rie^{i\theta}}{R^2 e^{i2\theta} + 1} d\theta \right| \leq \frac{R}{R^2-1} \int_0^{\pi} d\theta = \frac{\pi R}{R^2-1}$$

$$R \rightarrow \infty \implies \int_{\Gamma} \frac{1}{z^2+1} dz \rightarrow 0$$

$$\therefore \int_{-\infty}^{\infty} \frac{1}{x^2+1} dx = \frac{\pi}{\sqrt{2}}$$

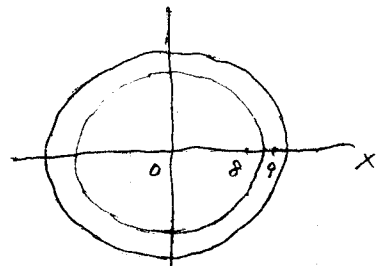
$$4.12 \quad R < 8 < r < \infty$$

$$\int_{C_R} \frac{dz}{(z-8)(z-9)} = 0$$

$$8 < R < 9 < \infty$$

$$\int_{C_R} \frac{dz}{(z-8)(z-9)} = 2\pi i \{ \text{Res}(8) \} \quad \text{Res}(8) = \lim_{z \rightarrow 8} \frac{1}{z-9} = -1$$

$$\therefore \int_{C_R} \frac{dz}{(z-8)(z-9)} = -2\pi i$$



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$$4.12 \quad R > 9 \text{ ok?} \quad \text{Res}(9) = \lim_{z \rightarrow 9} \frac{1}{z-8} = 1$$

$$\int_{CR} \frac{1}{(z-8)(z-9)} dz = 2\pi i (\text{Res}(8) + \text{Res}(9)) = 0$$

$$4.13 \quad \frac{1}{2\pi i} \int_C \frac{e^{ikz}}{z^2+9} dz = \text{Res}(3i)$$

$$\text{Res}(3i) = \lim_{z \rightarrow 3i} \frac{e^{ikz}}{z+3i} = \frac{e^{-3k}}{6i}$$

$$\int_C \frac{e^{ikz}}{z^2+9} dz = \frac{\pi e^{-3k}}{3}$$

$$\int_C \frac{e^{ikz}}{z^2+9} dz = \int_{-R}^0 \frac{e^{ikx}}{x^2+9} dx + \int_0^R \frac{e^{ikx}}{x^2+9} dx + \int_{\Gamma} \frac{e^{ikz}}{z^2+9} dz \quad \Gamma: |z|=R$$

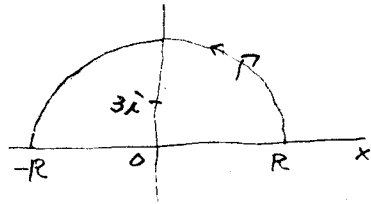
$$z = Re^{i\theta}$$

$$\left| \int_{\Gamma} \frac{e^{ikz}}{z^2+9} dz \right| = \left| \int_0^{2\pi} \frac{e^{ikR\cos\theta} e^{-kR\sin\theta}}{R^2 e^{i2\theta} + 9} R i e^{i\theta} d\theta \right|$$

$$\leq \frac{R}{R^2-9} \int_0^{\pi} e^{-kR\sin\theta} d\theta \leq \frac{R\pi}{R^2-9}$$

$$\therefore R \rightarrow \infty \text{ ok?} \quad \int_{\Gamma} \frac{e^{ikz}}{z^2+9} dz \rightarrow 0$$

$$\therefore \int_{-\infty}^{\infty} \frac{e^{ikx}}{x^2+9} dx = \frac{\pi e^{-3k}}{3}$$



P.80

§.5 正確率

$$5.1 (1) {}_4C_2 \left(\frac{1}{2}\right)^4 \cdot {}_3C_1 \left(\frac{1}{2}\right)^3 \cdot {}_2C_2 \left(\frac{1}{2}\right)^2 = \frac{4 \cdot 3}{2} \cdot \frac{3}{1} \cdot \frac{1}{2^9} = \frac{9}{2^8} = \frac{9}{256}$$

$$(2) (a, b, c) \quad a < c \leq b \leq 2$$

$$b=2, c=2, a=1, 0 \quad b=1, c=1, a=0$$

$$b=2, c=1, a=0$$

$$(1, 2, 2) \text{ の } k \text{ は } {}_4C_1 \left(\frac{1}{2}\right)^4 \cdot {}_3C_2 \left(\frac{1}{2}\right)^3 \cdot {}_2C_2 \left(\frac{1}{2}\right)^2 = \frac{3}{128}$$

$$(0, 2, 2) \text{ の } k \text{ は } {}_4C_0 \left(\frac{1}{2}\right)^4 \cdot {}_3C_2 \left(\frac{1}{2}\right)^3 \cdot {}_2C_2 \left(\frac{1}{2}\right)^2 = \frac{3}{512}$$

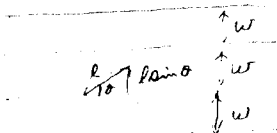
$$(0, 2, 1) \text{ の } k \text{ は } {}_4C_0 \left(\frac{1}{2}\right)^4 \cdot {}_3C_2 \left(\frac{1}{2}\right)^3 \cdot {}_2C_1 \left(\frac{1}{2}\right)^2 = \frac{3}{256}$$

$$(0, 1, 1) \text{ の } k \text{ は } {}_4C_0 \left(\frac{1}{2}\right)^4 \cdot {}_3C_1 \left(\frac{1}{2}\right)^3 \cdot {}_2C_1 \left(\frac{1}{2}\right)^2 = \frac{3}{256}$$

$$= \frac{3}{128} + \frac{3}{512} + \frac{3}{256} + \frac{3}{256} = \frac{27}{512}$$

$$(3) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 2 & 2-a & 2 & 2 \\ 3 & 3 & 3+b & 3 \\ 4 & 4 & 4 & 4-c \end{vmatrix} = 4 \quad \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & -c \end{vmatrix} = 4 \quad \therefore abc = 4$$

$$\therefore (a, b, c) = (4, 1, 1) \quad (2, 2, 1) \quad (2, 1, 2) \quad (1, 2, 2)$$

5.2 問題の $w < l$ を $w > l$ に訂正

針と直線が交差する正確率

$$\frac{l \sin \theta}{w}$$

$$\int_0^{\pi} \frac{l \sin \theta}{w} \frac{d\theta}{\pi} = \frac{l}{w\pi} [-\cos \theta]_0^{\pi} = \frac{2l}{w\pi}$$

$$5.3 \quad m, n \quad m^2 + 2n^2 \leq 72$$

$$(1) \quad m=5 \text{ の } k \text{ は } m=1, 2, 3, 4$$

$$m \leq 4 \text{ の } k \text{ は } m=1, 2, 3, 4, 5, 6$$

$$1 - \left(\frac{1}{8} + \frac{1}{36} + \frac{1}{36}\right) = 1 - \frac{2}{9} = \frac{7}{9}$$

$$m=6 \text{ の } k \text{ は } \frac{1}{8}$$

$$m=5, m=5 \quad \frac{1}{8} \times \frac{1}{8} = \frac{1}{36}$$

$$m=5, m=6 \quad \frac{1}{8} \times \frac{1}{8} = \frac{1}{36}$$

P.81

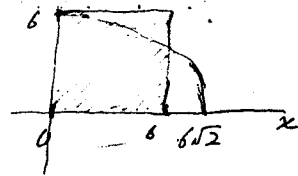
$$5.3 (2) \quad 0 \leq x \leq 6, 0 \leq y \leq 6 \quad x^2 + 2y^2 \leq 72$$

$$y = \frac{1}{\sqrt{2}} \sqrt{72 - x^2}$$

$$= 6 \sqrt{1 - \frac{x^2}{72}}$$

$$\frac{x^2}{72} + \frac{y^2}{36} \leq 1$$

$$\frac{x}{\sqrt{72}} = \sin t$$



$$\int_0^6 6 \sqrt{1 - \frac{x^2}{72}} dx = \int_0^{\frac{\pi}{4}} 6 \cos t \sqrt{72} \cos t dt = 36\sqrt{2} \int_0^{\frac{\pi}{4}} \frac{1 + \cos 2t}{2} dt$$

$$= 18\sqrt{2} \left[t + \frac{1}{2} \sin 2t \right]_0^{\frac{\pi}{4}} = 18\sqrt{2} \left(\frac{\pi}{4} + \frac{1}{2} \right) = \frac{9\sqrt{2}}{2} (\pi + 2)$$

$$\therefore \frac{1}{36} \cdot \frac{9\sqrt{2}}{2} (\pi + 2) = \frac{\sqrt{2}}{8} (\pi + 2)$$

5.4

1	0	送信
0.6	0.4	
0.9	0.95	受信

$$(1) \quad 0.6 \times 0.9 + 0.4 \times 0.05 = 0.56$$

$$(2) \quad \frac{0.6 \times 0.9}{0.6 \times 0.9 + 0.4 \times 0.05} = \frac{27}{28}$$

5.5 問題で A の袋青 4 個と打

	白	赤	青
A	8	3	4
B	4	7	4
C	7	3	5

白 \rightarrow A, 赤 \rightarrow B, 青 \rightarrow C

$$(1) \quad \begin{array}{l} 1 \quad 2 \quad 3 \\ A \rightarrow A \rightarrow A \\ \quad \searrow \rightarrow B \rightarrow A \\ \quad \quad \searrow \rightarrow C \rightarrow A \end{array}$$

$$\left(\frac{8}{15}\right)^3$$

$$\frac{3}{15} \cdot \frac{4}{15}$$

$$\frac{4}{15} \cdot \frac{7}{15}$$

$$P_{A_3} = \frac{64 + 12 + 28}{15^2} = \frac{104}{225}$$

$$(2) \quad \begin{pmatrix} P_{A_n} \\ P_{B_n} \\ P_{C_n} \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^{n-1} \begin{pmatrix} P_{A_1} \\ P_{B_1} \\ P_{C_1} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{8}{15} & \frac{4}{15} & \frac{7}{15} \\ \frac{3}{15} & \frac{7}{15} & \frac{3}{15} \\ \frac{4}{15} & \frac{4}{15} & \frac{5}{15} \end{pmatrix} \begin{pmatrix} P_{A_1} \\ P_{B_1} \\ P_{C_1} \end{pmatrix}$$

P. 81

5.50)

$$\begin{vmatrix} 8-\lambda & 4 & 7 \\ 3 & 7-\lambda & 3 \\ 4 & 4 & 5-\lambda \end{vmatrix} = 0$$

$$-\lambda^3 + 20\lambda^2 - 79\lambda + 60 = 0$$

$$\lambda^3 - 20\lambda^2 + 79\lambda - 60 = 0$$

$$(\lambda - 1)(\lambda - 4)(\lambda - 15) = 0$$

$$\begin{array}{r|rrrr} 1 & -20 & 79 & -60 & \\ \hline & 1 & -19 & 60 & \\ \hline & & 4 & -60 & \\ \hline & & & 1 & -15 & 0 \end{array}$$

$\lambda = 1$ の固有ベクトル

$$\begin{pmatrix} 7 & 4 & 7 \\ 3 & 6 & 3 \\ 4 & 4 & 4 \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$\lambda = 4$ の固有ベクトル

$$\begin{pmatrix} 4 & 4 & 7 \\ 3 & 3 & 3 \\ 4 & 4 & 1 \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$\lambda = 15$ の固有ベクトル

$$\begin{pmatrix} -7 & 4 & 7 \\ 3 & -8 & 3 \\ 4 & 4 & -10 \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 34 \\ 21 \\ 22 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 1 & 34 \\ 0 & -1 & 21 \\ -1 & 0 & 22 \end{pmatrix} \quad A = \begin{pmatrix} \frac{8}{15} & \frac{4}{15} & \frac{7}{15} \\ \frac{3}{15} & \frac{7}{15} & \frac{3}{15} \\ \frac{4}{15} & \frac{4}{15} & \frac{5}{15} \end{pmatrix} \quad \text{対角化}$$

$$P^{-1}AP = \begin{pmatrix} \frac{1}{15} & 0 & 0 \\ 0 & \frac{4}{15} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad A^n = P \begin{pmatrix} (\frac{1}{15})^n & 0 & 0 \\ 0 & (\frac{4}{15})^n & 0 \\ 0 & 0 & 1 \end{pmatrix} P^{-1}$$

$n \rightarrow \infty$ の場合

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} P^{-1}$$

$$P^{-1} = \begin{pmatrix} \frac{22}{77} & \frac{22}{77} & \frac{55}{77} \\ \frac{21}{77} & -\frac{56}{77} & \frac{21}{77} \\ \frac{1}{77} & \frac{1}{77} & \frac{1}{77} \end{pmatrix}$$

P^{-1}

$$\begin{array}{r|rrrr} 1 & 1 & 34 & 1 & 0 & 0 \\ 0 & -1 & 21 & 0 & 1 & 0 \\ \hline -1 & 0 & 22 & 0 & 0 & 1 \\ 1 & 1 & 34 & 1 & 0 & 0 \\ 0 & -1 & 21 & 0 & 1 & 0 \\ 0 & 1 & 56 & 1 & 0 & 1 \\ \hline 1 & 0 & 55 & 1 & 1 & 0 \\ 0 & -1 & 21 & 0 & 1 & 0 \\ 0 & 0 & 77 & 1 & 1 & 1 \\ \hline 1 & 0 & 0 & \frac{22}{77} & \frac{22}{77} & -\frac{55}{77} \\ 0 & -1 & 0 & -\frac{21}{77} & \frac{56}{77} & -\frac{21}{77} \\ 0 & 0 & 1 & \frac{1}{77} & \frac{1}{77} & \frac{1}{77} \end{array}$$

P. 81

$$\begin{aligned}
 P \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} P^{-1} &= \begin{pmatrix} 1 & 1 & 34 \\ 0 & -1 & 21 \\ -1 & 0 & 22 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} P^{-1} \\
 &= \begin{pmatrix} 0 & 0 & 34 \\ 0 & 0 & 21 \\ 0 & 0 & 22 \end{pmatrix} \begin{pmatrix} \frac{22}{77} & \frac{22}{77} & -\frac{55}{77} \\ \frac{21}{77} & -\frac{55}{77} & \frac{21}{77} \\ \frac{1}{77} & \frac{1}{77} & \frac{1}{77} \end{pmatrix} = \begin{pmatrix} \frac{34}{77} & \frac{34}{77} & \frac{34}{77} \\ \frac{22}{77} & \frac{22}{77} & \frac{22}{77} \\ \frac{21}{77} & \frac{21}{77} & \frac{21}{77} \end{pmatrix}
 \end{aligned}$$

$$\therefore \frac{34}{77}$$

5.6 白 4, 黑 1, 3回 2 白 1 2 75% $(\frac{1}{5})^3 = \frac{1}{125}$

$$\therefore 1 - \frac{1}{125} = \frac{124}{125}$$

5.7 1か6が出る確率 $\frac{1}{3}$

$$A \text{ が } 4 \text{ 回} \quad {}_5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right) = 5 \cdot \frac{2}{3^5} = \frac{10}{3^5}$$

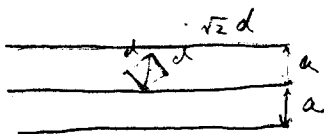
$$A \text{ が } 5 \text{ 回} \quad {}_5C_5 \left(\frac{1}{3}\right)^5 = \frac{1}{3^5}$$

$$\therefore \frac{1}{243} + \frac{10}{243} = \frac{11}{243}$$

5.8 当り 確率 $\frac{3}{10}$

$$\frac{7}{10} \cdot \frac{6}{9} \cdot \frac{3}{8} = \frac{17}{40}$$

5.9



$$\sqrt{2}d < a$$

$$\begin{aligned}
 \frac{d \sin \alpha}{a} \cdot \frac{d\alpha}{\pi} & \quad 2) \int_0^{\pi} \frac{d \sin \alpha}{a} \frac{d\alpha}{\pi} = \frac{2d}{a\pi} [-\cos \alpha]_0^{\pi} \\
 & = \frac{4d}{a\pi}
 \end{aligned}$$

5.10 白 $2N$ 赤 1

$$(1) P_{1,N} = {}_N C_1 \frac{(2N)^{N-1}}{(2N+1)^{N-1}} \frac{1}{2N+1} = \frac{(2N)^N}{2(2N+1)^N}$$

$$\begin{aligned}
 (2) \lim_{N \rightarrow \infty} P_{1,N} &= \lim_{N \rightarrow \infty} \frac{1}{2} \frac{1}{\left(1 + \frac{1}{2N}\right)^N} = \frac{1}{2} \lim_{N \rightarrow \infty} \frac{1}{\left(1 + \frac{1}{2N}\right)^{2N} \frac{1}{2}} \\
 &= \frac{1}{2\sqrt{e}}
 \end{aligned}$$

P. 81.82

$$5.10 (3) \quad \frac{P_{m+1, N}}{P_{m, N}} = \frac{{}^N C_{m+1} \frac{(2N)^{N-m-1}}{(2N+1)^N}}{{}^N C_m \frac{(2N)^{N-m}}{(2N+1)^N}} = \frac{\frac{N!}{(m+1)!(N-m-1)!} \cdot \frac{1}{2N}}{\frac{N!}{m!(N-m)!}}$$

$$= \frac{N-m}{m+1} \cdot \frac{1}{2N} = \frac{N-m}{2N(m+1)}$$

$$(4) \quad \lim_{N \rightarrow \infty} \frac{P_{m+1, N}}{P_{m, N}} = \lim_{N \rightarrow \infty} \frac{N-m}{2N(m+1)} = \frac{1}{2(m+1)}$$

$$5.11 (1) \quad \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$(2) \quad 1 - \left(\frac{7}{8}\right)^8 = \frac{1695}{4096}$$

5.12 抽 3 自 7

$$\frac{{}_3 C_1 \times {}_2 C_1}{{}_{10} C_2} = \frac{3 \cdot 2}{\frac{10 \cdot 9}{2}} = \frac{7}{15}$$

5.13 抽 3 自 3

$$\frac{{}_3 C_2}{{}_6 C_2} = \frac{3 \cdot 2}{6 \cdot 5} = \frac{1}{5}$$

P.82

§ 6. 期待値と分散

6.1 20面体 12面赤, 8面青 4回振る.

赤 a 回青 x 回 $a \geq x$ のとき a 点 $a < x$ のとき 0 点

$$a=4 \text{ のとき } 4 \cdot C_4 \left(\frac{3}{5}\right)^4 = \frac{324}{5^4}$$

$$a=3 \text{ のとき } 3 \cdot C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right) = \frac{648}{5^4}$$

$$a=2 \text{ のとき } 2 \cdot C_2 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^2 = \frac{732}{5^4}$$

 $a=3$ のとき最大となり, そのときの期待値は $\frac{648}{625}$
6.2 (1) $P(X_1 = x_{1i}) = P_{1i}$, $P(X_2 = x_{2j}) = P_{2j}$ とすると $\sum_{i=1}^n P_{1i} = 1$, $\sum_{j=1}^m P_{2j} = 1$,

$$E(X_1 + X_2) = \sum_{i=1}^n \sum_{j=1}^m (x_{1i} + x_{2j}) \cdot P_{1i} \cdot P_{2j} = \sum_{i=1}^n \sum_{j=1}^m x_{1i} P_{1i} P_{2j} + \sum_{j=1}^m \sum_{i=1}^n x_{2j} P_{2j} P_{1i}$$

$$= \sum_{i=1}^n x_{1i} P_{1i} \sum_{j=1}^m P_{2j} + \sum_{j=1}^m x_{2j} P_{2j} \sum_{i=1}^n P_{1i}$$

$$= \sum_{i=1}^n x_{1i} P_{1i} + \sum_{j=1}^m x_{2j} P_{2j} = E(X_1) + E(X_2)$$

$$\therefore E(X_1 + X_2) = E(X_1) + E(X_2)$$

$$(2) E(X) = 1 \times \frac{1}{2} + 2 \times \frac{1}{2} + 3 \times \frac{1}{2} + 4 \times \frac{1}{2} + 5 \times \frac{1}{2} + 6 \times \frac{1}{2} = \frac{7}{2}$$

$$E(X_1 + X_2 + \dots + X_n) = E(X_1) + E(X_2) + \dots + E(X_n) \\ = n \cdot \frac{7}{2} = \frac{7}{2}n$$

6.3 (1) A, B の二人が勝負する. 0: A の勝, x : A の負

$$0 \rightarrow x \rightarrow 0 \rightarrow \dots$$

$$2 \cdot \left(\frac{1}{2}\right)^2 + 3 \cdot \left(\frac{1}{2}\right)^3 + 4 \cdot \left(\frac{1}{2}\right)^4 + \dots + n \cdot \left(\frac{1}{2}\right)^n + \dots$$

$$S_n = 2 \cdot \left(\frac{1}{2}\right)^2 + 3 \cdot \left(\frac{1}{2}\right)^3 + \dots + (n-1) \cdot \left(\frac{1}{2}\right)^{n-1} + n \cdot \left(\frac{1}{2}\right)^n$$

$$\frac{1}{2} S_n = 2 \cdot \left(\frac{1}{2}\right)^3 + 3 \cdot \left(\frac{1}{2}\right)^4 + \dots + (n-1) \cdot \left(\frac{1}{2}\right)^n + n \cdot \left(\frac{1}{2}\right)^{n+1}$$

$$\frac{1}{2} S_n = 2 \cdot \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \dots + \left(\frac{1}{2}\right)^n + n \cdot \left(\frac{1}{2}\right)^{n+1}$$

$$= \left(\frac{1}{2}\right)^2 + \frac{\left(\frac{1}{2}\right)^2 (1 - \left(\frac{1}{2}\right)^{n-1})}{1 - \frac{1}{2}} + n \cdot \left(\frac{1}{2}\right)^{n+1}$$

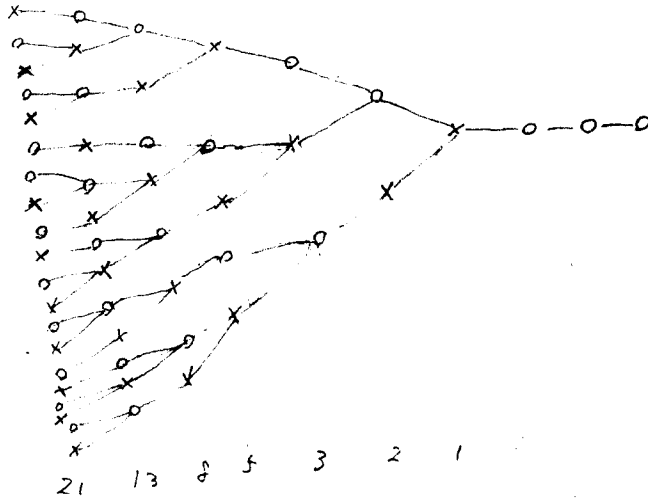
$$S_n = \frac{1}{2} + 1 - \left(\frac{1}{2}\right)^{n-1} - n \cdot \left(\frac{1}{2}\right)^n$$

$$\therefore \lim_{n \rightarrow \infty} S_n = \frac{3}{2}$$

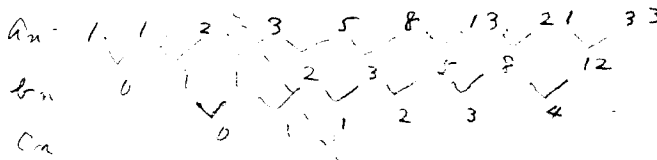
A, B のとき $\frac{3}{2}$ かが勝つ $2 \times \frac{3}{2} = 3$ 期待値

P. P2

(2) 樹形圖



$$S = 3 \cdot \left(\frac{1}{2}\right)^3 + 4 \cdot \left(\frac{1}{2}\right)^4 + 5 \cdot 2 \left(\frac{1}{2}\right)^5 + 6 \cdot 3 \left(\frac{1}{2}\right)^6 + 7 \cdot 5 \left(\frac{1}{2}\right)^7 + 8 \cdot 2 \left(\frac{1}{2}\right)^8 + 9 \cdot 13 \left(\frac{1}{2}\right)^9 + 10 \cdot 21 \left(\frac{1}{2}\right)^{10} + 11 \cdot 33 \left(\frac{1}{2}\right)^{11} + \dots$$



$$b_n = a_{n+1} - a_n \quad c_n = b_{n+1} - b_n \quad \& \# \cdot < k$$

$$c_n = m \quad \therefore b_{n+1} - b_n = \frac{n(n+1)}{2} \quad \therefore b_n = \frac{n^2 - n + 4}{2}$$

$$a_n - a_1 = \sum_{k=1}^{n-1} \frac{k^2 - k + 4}{2} = \frac{1}{2} \left\{ \frac{(n-1)n(2n-1)}{6} - \frac{(n-1)n}{2} + 4(n-1) \right\}$$

$$= \frac{1}{6} \{ (n-1)^3 + 11(n-1) \}$$

$$\therefore a_n = \frac{1}{6} \{ (n-1)^3 + 11(n-1) + 18 \}$$

$$S_n = \frac{1}{2} + 2 \left(\frac{1}{2}\right)^2 + 3 \left(\frac{1}{2}\right)^3 + \dots + n \left(\frac{1}{2}\right)^n \quad \& \# \cdot < k$$

$$\frac{1}{2} S_n = \left(\frac{1}{2}\right)^2 + 2 \left(\frac{1}{2}\right)^3 + \dots + n \left(\frac{1}{2}\right)^{n+1}$$

$$S_n = 2 \left\{ \sum_{k=1}^n \left(\frac{1}{2}\right)^k + n \left(\frac{1}{2}\right)^{n+1} \right\} = 2 \left\{ 1 - \left(\frac{1}{2}\right)^n + n \left(\frac{1}{2}\right)^{n+1} \right\}$$

$$= 2 - \left(\frac{1}{2}\right)^{n-1} - n \left(\frac{1}{2}\right)^n$$

$$\lim S_n = 2$$

P 82

$$A_n = \frac{1}{2} + 2^2 \left(\frac{1}{2}\right)^2 + 3^2 \left(\frac{1}{2}\right)^3 + 4^2 \left(\frac{1}{2}\right)^4 + \dots + n^2 \left(\frac{1}{2}\right)^n$$

$$\frac{1}{2} A_n = \left(\frac{1}{2}\right)^2 + 2^2 \left(\frac{1}{2}\right)^3 + 3^2 \left(\frac{1}{2}\right)^4 + \dots + n^2 \left(\frac{1}{2}\right)^{n+1}$$

$$\frac{1}{2} A_n = \sum_{k=1}^n (2k-1) \left(\frac{1}{2}\right)^k - n^2 \left(\frac{1}{2}\right)^{n+1} = 2 \cdot \left(2 - \left(\frac{1}{2}\right)^n - n \left(\frac{1}{2}\right)^n\right) - 1 + \left(\frac{1}{2}\right)^n - n^2 \left(\frac{1}{2}\right)^{n+1}$$

$$= 3 - \left(\frac{1}{2}\right)^{n-2} - n \left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{2}\right)^n - n^2 \left(\frac{1}{2}\right)^{n+1}$$

$$A_n = 6 - \left(\frac{1}{2}\right)^{n-3} - n \left(\frac{1}{2}\right)^{n-2} + \left(\frac{1}{2}\right)^{n-1} - n^2 \left(\frac{1}{2}\right)^n$$

$$\therefore \lim_{n \rightarrow \infty} A_n = 6$$

同様に

$$x_n = \frac{1}{2} + 2^3 \left(\frac{1}{2}\right)^2 + 3^3 \left(\frac{1}{2}\right)^3 + 4^3 \left(\frac{1}{2}\right)^4 + \dots + n^3 \left(\frac{1}{2}\right)^n \quad \& a; < \infty$$

$$\frac{1}{2} x_n = \left(\frac{1}{2}\right)^2 + 2^3 \left(\frac{1}{2}\right)^3 + 3^3 \left(\frac{1}{2}\right)^4 + \dots + n^3 \left(\frac{1}{2}\right)^{n+1}$$

$$= \sum (3k^2 - 3k + 1) \left(\frac{1}{2}\right)^k - n^3 \left(\frac{1}{2}\right)^{n+1}$$

$$= 3 \sum k^2 \left(\frac{1}{2}\right)^k - 3 \sum k \left(\frac{1}{2}\right)^k + \sum \left(\frac{1}{2}\right)^k - n^3 \left(\frac{1}{2}\right)^{n+1}$$

$$x_n = 6 \sum k^2 \left(\frac{1}{2}\right)^k - 6 \sum k \left(\frac{1}{2}\right)^k + 2 \sum \left(\frac{1}{2}\right)^k - n^3 \left(\frac{1}{2}\right)^n$$

$$\lim_{n \rightarrow \infty} x_n = 36 - 12 + 2 = 26$$

$$T_n = \frac{1}{2} + 2^4 \left(\frac{1}{2}\right)^2 + 3^4 \left(\frac{1}{2}\right)^3 + 4^4 \left(\frac{1}{2}\right)^4 + \dots + n^4 \left(\frac{1}{2}\right)^n \quad \& a; < \infty$$

$$T_n = 2 \left[\sum 4k^3 \left(\frac{1}{2}\right)^k - \sum 6k^2 \left(\frac{1}{2}\right)^k + 4 \sum k \left(\frac{1}{2}\right)^k - \sum \left(\frac{1}{2}\right)^k \right] - n^4 \left(\frac{1}{2}\right)^n$$

$$\lim_{n \rightarrow \infty} T_n = 8 \times 26 - 12 \times 6 + 8 \times 2 - 2 = 150$$

$$S = 3 \cdot \left(\frac{1}{2}\right)^3 + 4 \cdot \left(\frac{1}{2}\right)^4 + 5 \cdot 2 \left(\frac{1}{2}\right)^5 + 6 \cdot 3 \left(\frac{1}{2}\right)^6 + 7 \cdot 5 \left(\frac{1}{2}\right)^7 + 8 \cdot 8 \left(\frac{1}{2}\right)^8 + 9 \cdot 13 \left(\frac{1}{2}\right)^9$$

$$= \frac{3}{8} + \frac{1}{4} + \frac{5}{16} + \sum_{k=0}^6 (k+6) \frac{(k^2+11k+18)}{6} \left(\frac{1}{2}\right)^{k+6}$$

$$= \frac{15}{16} + \frac{1}{6} \left(\frac{1}{2}\right)^6 \cdot \sum_{k=0}^6 (k^2+6k^2+11k^2+84k+108) \left(\frac{1}{2}\right)^k$$

$$= \frac{15}{16} + \frac{1}{6 \times 64} (150 + 6 \times 26 + 11 \times 6 + 84 \times 2 + 108 \times 2)$$

$$= \frac{15}{16} + \frac{63}{32} = \frac{93}{32}$$

$$\therefore \text{期待値は } \frac{93}{16}$$

$$6.4 \quad (1) \quad \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$(2) \quad 0 \text{ 匹} \quad \left(\frac{7}{8}\right)^3 = \frac{343}{8^3} \quad 1 \text{ 匹} \quad 3C_1 \left(\frac{7}{8}\right)^2 \frac{1}{8} = \frac{147}{8^3}$$

$$2 \text{ 匹} \quad 3C_2 \left(\frac{7}{8}\right) \left(\frac{1}{8}\right)^2 = \frac{21}{8^3} \quad 3 \text{ 匹} \quad 3C_3 \left(\frac{1}{8}\right)^3 = \frac{1}{8^3}$$

$$\text{平均} \quad 0 \times \frac{343}{8^3} + 1 \times \frac{147}{8^3} + 2 \times \frac{21}{8^3} + 3 \times \frac{1}{8^3} = \frac{9}{8}$$

$$\text{分散} \quad \frac{1}{8^3} (3^2 \times 343 + 5^2 \times 147 + 13^2 \times 21 + 21^2)$$

$$= \frac{1}{8^3} \times 10752 = \frac{21}{8^2} = \frac{21}{64}$$

$$(3) \quad nC_r \left(\frac{1}{8}\right)^r \left(\frac{7}{8}\right)^{n-r}$$

$$6.5 \quad (1) \quad 1 \times \frac{1}{100} + 2 \times \frac{99}{100} \cdot \frac{1}{99} + 3 \times \frac{99}{100} \cdot \frac{98}{99} \cdot \frac{1}{98} + \dots + 100 \times \frac{1}{100}$$

$$= \frac{1}{100} (1 + 2 + 3 + \dots + 100) = \frac{1}{100} \cdot \frac{100 \times 101}{2} = 50.5$$

(2) 100 人目

$$6.6 \quad (1) \quad X \text{ が } \bar{x} = \frac{2}{5}, \quad X \text{ が } \bar{x} \text{ と } t \text{ (} \frac{3}{5} \text{)}$$

$$P(0) = \left(\frac{3}{5}\right)^3 = \frac{27}{125} \quad P(1) = 3 \frac{2}{5} \cdot \left(\frac{3}{5}\right)^2 = \frac{54}{125}$$

$$P(2) = 3 \left(\frac{2}{5}\right)^2 \frac{3}{5} = \frac{36}{125} \quad P(3) = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$$

$$(2) \quad E(X) = \frac{1}{125} (0 \times 27 + 1 \times 54 + 2 \times 36 + 3 \times 8) = \frac{6}{5}$$

$$V(X) = \frac{1}{125} (6^2 \times 27 + 1^2 \times 54 + 4^2 \times 36 + 9^2 \times 8) \frac{1}{5^2}$$

$$= \frac{40}{125} = \frac{18}{25} = 0.72$$

$$6.7 \quad \begin{array}{ccc} \text{赤} & 3 & \text{白} & 2 \\ \text{0} & & \text{x} & \text{0} \\ & & \text{x} & \text{0} \end{array}$$

$$(1) \quad \frac{3}{5} \times \frac{2}{4} = \frac{3}{10} \quad \frac{2}{5} \times \frac{3}{4} = \frac{3}{10} \quad \therefore \frac{3}{5}$$

$$(2) \quad E(X) = 0 \times \frac{2}{5} + 1 \times \frac{3}{5} = \frac{3}{5}$$

$$V(X) = \left(\frac{3}{5}\right)^2 \times \frac{2}{5} + \left(\frac{2}{5}\right)^2 \times \frac{3}{5} = \frac{6}{25}$$

P. 82 83

§. 7 正規率関数

7.1

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad N(\mu, \sigma^2)$$

$$(1) \quad \frac{x-\mu}{\sigma} = t \text{ とおくと } dx = \sigma dt$$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \quad \frac{t}{\sqrt{2}} = z$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} dz = 1 \quad \therefore \text{これは } N(0, 1) \text{ の積分}$$

$$(2) \quad N = 500 \quad \bar{x} = 151, \quad D = 15$$

$$\therefore f(x) = \frac{1}{15\sqrt{2\pi}} e^{-\frac{(x-151)^2}{2 \times 15}} \quad \frac{x-151}{15} = t$$

$$\begin{aligned} \int_{145}^{154} f(x) dx &= \int_{-0.4}^{0.2} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \\ &= \int_0^{0.4} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt + \int_0^{0.2} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \\ &= 0.1554 + 0.0793 = 0.2347 \end{aligned}$$

$$500 \times 0.2347 = 117.35 \quad \approx \underline{117}$$

7.2

$$f(x) = \begin{cases} m e^{-mx} & x \geq 0 \quad (m > 0) \\ 0 & x < 0 \end{cases}$$

$$(1) \quad F(x) = \int_0^x m e^{-mt} dt = [-e^{-mt}]_0^x = 1 - e^{-mx}$$

$$F(x) = \begin{cases} 1 - e^{-mx} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\begin{aligned} \mu &= \int_{-\infty}^{\infty} x f(x) dx = \int_0^{\infty} m x e^{-mx} dx = [-x e^{-mx}]_0^{\infty} + \int_0^{\infty} e^{-mx} dx \\ &= [-\frac{1}{m} e^{-mx}]_0^{\infty} = \frac{1}{m} \end{aligned}$$

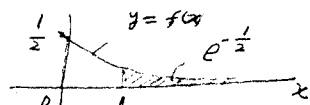
$$\begin{aligned} \sigma^2 &= \int_{-\infty}^{\infty} (x - \frac{1}{m})^2 m e^{-mx} dx = [-(x - \frac{1}{m})^2 e^{-mx}]_0^{\infty} + 2 \int_0^{\infty} (x - \frac{1}{m}) e^{-mx} dx \\ &= \frac{1}{m^2} - 2 \left[\frac{1}{m} (x - \frac{1}{m}) e^{-mx} \right]_0^{\infty} + 2 \frac{1}{m} \int_0^{\infty} e^{-mx} dx = \frac{1}{m^2} \end{aligned}$$

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§. 8 統計

7.2 (3) 平均 2.5) $\mu = \frac{1}{m} = 2 \quad \therefore m = \frac{1}{2} \quad f(x) = \frac{1}{2} e^{-\frac{1}{2}x} \quad x \geq 0$

(1) (5) $F(x) = \begin{cases} 1 - e^{-\frac{1}{2}x} & x > 0 \\ 0 & x < 0 \end{cases}$



$\therefore F(1) = 1 - e^{-\frac{1}{2}}$

平均 $\int_0^{\infty} (x-1) \frac{1}{2} e^{-\frac{1}{2}x} dx = [- (x-1) e^{-\frac{1}{2}x}]_0^{\infty} + \int_0^{\infty} e^{-\frac{1}{2}x} dx$
 $= [-2 e^{-\frac{1}{2}x}]_0^{\infty} = 2 e^{-\frac{1}{2}}$

7.3 $P(X=k) = e^{-\lambda} \frac{\lambda^k}{k!}$

$\mu = E(X) = \sum k e^{-\lambda} \frac{\lambda^k}{k!} = \lambda \sum e^{-\lambda} \frac{\lambda^{k-1}}{(k-1)!} = \lambda e^{-\lambda} e^{\lambda} = \lambda$

$\sigma^2 = V(X) = \sum (k-\lambda)^2 e^{-\lambda} \frac{\lambda^k}{k!}$

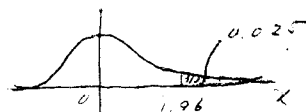
$= \sum e^{-\lambda} \frac{k^2 - 2\lambda k + \lambda^2}{k!} \lambda^k = \lambda^2 \sum e^{-\lambda} \frac{\lambda^k}{k!} - 2\lambda^2 \sum e^{-\lambda} \frac{\lambda^{k-1}}{(k-1)!} + \sum e^{-\lambda} \frac{k \lambda^k}{k!}$

$= \lambda^2 - 2\lambda^2 + \sum e^{-\lambda} \frac{k(k-1) + k}{k!} \lambda^k$

$= -\lambda^2 + \lambda^2 \sum e^{-\lambda} \frac{\lambda^{k-2}}{(k-2)!} + \lambda \sum e^{-\lambda} \frac{\lambda^{k-1}}{(k-1)!} = \lambda$

8.1

$\frac{1}{\sqrt{2\pi}} \int_0^{1.96} e^{-\frac{x^2}{2}} dx = 0.475$



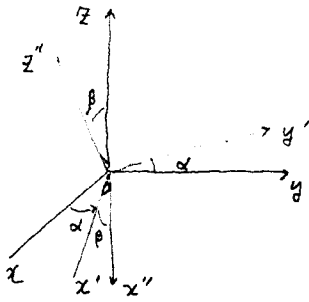
$\frac{|\bar{x} - \mu_0|}{\sigma/\sqrt{n}} \geq 1.96 \quad 0.4 \geq 1.96 \frac{\sigma}{\sqrt{n}} \quad \sigma = 4$

$\sqrt{n} \geq 1.96 \frac{4}{0.4} = 19.6 \quad n \geq 19.6^2 = 384.16$

$n \geq 385$

§.9 ベクトル解析

9.1

(1) S'' 系で $(1, 0, 0)$ は S' 系で $(\cos\beta, 0, -\sin\beta)$ S 系で $(\cos\alpha\cos\beta, \sin\alpha\cos\beta, -\sin\beta)$

$$\therefore \cos\theta = \cos\alpha\cos\beta$$

$$(2) \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha\cos\beta & \sin\alpha\cos\beta & \sin\beta \\ -\sin\alpha & \cos\alpha & 0 \\ -\cos\alpha\sin\beta & -\sin\alpha\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$x'' = x\cos\alpha\cos\beta + y\sin\alpha\cos\beta + z\sin\beta$$

$$y'' = -x\sin\alpha + y\cos\alpha$$

$$z'' = -x\cos\alpha\sin\beta - y\sin\alpha\sin\beta + z\cos\beta$$

(3) S 系で $(1, 1, 0)$ は S'' 系で $(\cos\alpha\cos\beta + \sin\alpha\sin\beta, -\sin\alpha + \cos\alpha, -\cos\alpha\sin\beta - \sin\alpha\cos\beta)$ S' 系で $(1, 1, 0)$ は S'' 系で $(\cos\beta, 1, -\sin\beta)$

$$\therefore \text{四面体 } OABC = \frac{1}{6} \begin{vmatrix} \cos\alpha + \sin\alpha & \cos\beta & -\sin\alpha + \cos\alpha & -(\cos\alpha + \sin\alpha)\sin\beta \\ \cos\beta & 1 & 1 & -\sin\beta \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

$$= \frac{1}{6} \begin{vmatrix} 0 & -2\sin\alpha & 0 \\ \cos\beta & 1 & -\sin\beta \\ 1 & 1 & 0 \end{vmatrix} = \frac{1}{3} |\sin\alpha\sin\beta|$$

9.2 $\vec{A} = (a_1, a_2, a_3)$ $\vec{B} = (b_1, b_2, b_3)$ $\vec{C} = (c_1, c_2, c_3)$ とおくと

$$\vec{B} \times \vec{C} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (b_2 c_3 - c_2 b_3) \vec{i} - (b_1 c_3 - b_3 c_1) \vec{j} + (b_1 c_2 - b_2 c_1) \vec{k}$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ (b_2 c_3 - c_2 b_3) & -(b_1 c_3 - b_3 c_1) & (b_1 c_2 - b_2 c_1) \end{vmatrix}$$

$$= (a_2 |b_1 c_2| + a_3 |c_1 b_3|) \vec{i} + (a_3 |b_2 c_3| - a_1 |b_1 c_3|) \vec{j} + (a_1 |b_1 c_3| + a_2 |b_2 c_3|) \vec{k}$$

同様にして

$$\vec{B} \times (\vec{C} \times \vec{A}) = (b_2 |c_1 c_2| + b_3 |c_1 c_3|) \vec{i} + (b_3 |c_2 c_3| - b_1 |c_1 c_3|) \vec{j} + (b_1 |c_1 c_3| + b_2 |c_2 c_3|) \vec{k}$$

$$\vec{C} \times (\vec{A} \times \vec{B}) = (c_2 |a_1 a_2| + c_3 |a_1 a_3|) \vec{i} + (c_3 |a_2 a_3| - c_1 |a_1 a_3|) \vec{j} + (c_1 |a_1 a_3| + c_2 |a_2 a_3|) \vec{k}$$

$$\vec{i} \text{ の成分 } a_2 b_1 c_2 - a_2 b_2 c_1 + a_3 b_1 c_3 - a_3 b_3 c_1 + a_2 b_2 c_1 - a_1 b_2 c_2 + a_3 b_3 c_1 - a_1 b_3 c_3$$

$$+ a_1 b_2 c_2 - a_2 b_1 c_2 + a_1 b_3 c_3 - a_3 b_1 c_3 = 0$$

$$\text{同様にして } \vec{j} \text{ の成分 } = 0 \quad \vec{k} = 0$$

$$\therefore \vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0$$

9.3 $\vec{a} = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}$ $\vec{c} = \begin{pmatrix} 0 \\ 0 \\ c \end{pmatrix}$

(1) $\vec{r} - \vec{a} = x(\vec{b} - \vec{a}) + y(\vec{c} - \vec{a})$

$$\therefore \vec{r} = (1-x-y) \vec{a} + x \vec{b} + y \vec{c} \quad 1-x-y = \lambda \quad x = \mu \quad y = \nu \quad \lambda + \mu + \nu = 1$$

$$\vec{r} = \lambda \vec{a} + \mu \vec{b} + \nu \vec{c} \quad \lambda + \mu + \nu = 1$$

(2) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \lambda a \\ \mu b \\ \nu c \end{pmatrix} \quad \therefore \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = \lambda + \mu + \nu = 1$

$$\therefore f(x, y, z) = \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

(3) $\text{grad } f = \left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \right)$

$$\vec{n} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \text{grad } f = \frac{abc}{\sqrt{a^2 b^2 + b^2 c^2 + c^2 a^2}} \text{grad } f$$

P 84

$$(4) \vec{S} = \frac{1}{2} (\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = \frac{1}{2} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = \frac{1}{2} (bc\vec{i} + ac\vec{j} + a^2\vec{k})$$

$$|\vec{S}| = \frac{1}{2} \sqrt{b^2c^2 + c^2a^2 + a^2b^2}$$

$\equiv 27^\circ \text{リント}$

$$9.4 \quad \vec{r} = \cos \omega t \vec{i} + \sin \omega t \vec{j}$$

$$(1) \frac{d\vec{r}}{dt} = -\omega \sin \omega t \vec{i} + \omega \cos \omega t \vec{j} = \vec{v}$$

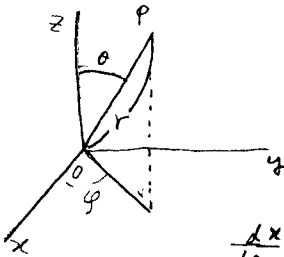
$$\therefore \vec{r} \cdot \vec{v} = \omega (-\cos \omega t \sin \omega t + \sin \omega t \cos \omega t) = 0 \quad \therefore \vec{r} \perp \vec{v}$$

$$(2) \frac{d\vec{v}}{dt} = -\omega^2 (\cos \omega t \vec{i} + \sin \omega t \vec{j})$$

$$\therefore \vec{a} = -\omega^2 \vec{r} \quad \therefore \vec{a} \text{ は } \vec{r} \text{ と反対の向き, 大きさは } \omega^2$$

$$(3) \vec{r} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos \omega t & \sin \omega t & 0 \\ -\omega \sin \omega t & \omega \cos \omega t & 0 \end{vmatrix} = \omega \vec{k} \quad (\text{一定})$$

9.5



$$x = r \sin \theta \cos \varphi$$

$$y = r \sin \theta \sin \varphi$$

$$z = r \cos \theta$$

$$r = r(s), \quad \theta = \theta(s), \quad \varphi = \varphi(s) \quad k \leq s \leq k$$

$$\frac{dx}{ds} = r' \sin \theta \cos \varphi + \theta' r \cos \theta \cos \varphi - \varphi' r \sin \theta \sin \varphi$$

$$\frac{dy}{ds} = r' \sin \theta \sin \varphi + \theta' r \cos \theta \sin \varphi + \varphi' r \sin \theta \cos \varphi$$

$$\frac{dz}{ds} = r' \cos \theta - \theta' r \sin \theta$$

$$\therefore |\vec{v}| = \sqrt{(r')^2 + (\theta' r)^2 + (\varphi' r \sin \theta)^2} \quad k \leq s \leq k$$

$$\cos \alpha = \frac{r'}{|\vec{v}|}, \quad \cos \beta = \frac{\theta' r}{|\vec{v}|}, \quad \cos \gamma = \frac{\varphi' r \sin \theta}{|\vec{v}|}$$

P 84

$$9.6 \quad P(t) = (t, t^2, t^3)$$

$$P(0) = (0, 0, 0), \quad P(a) = (a, a^2, a^3), \quad P(b) = (b, b^2, b^3)$$

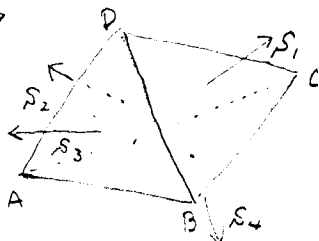
$$P(c) = (c, c^2, c^3) \quad (0 < a < b < c)$$

$$\vec{P}(a) \times \vec{P}(b) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & a^2 & a^3 \\ b & b^2 & b^3 \end{vmatrix} = a^2 b^3 (b-a) \vec{i} + ab(a^2 b^3) \vec{j} + ab(b-a) \vec{k}$$

$$\begin{aligned} \vec{P}(a) \times \vec{P}(b) \cdot \vec{P}(c) &= a^2 b^3 (b-a) + ab c^2 (a^2 b^3) + ab c^3 (b-a) \\ &= ab c (b-a) \{ ab - ca - (b + c^2) \} \\ &= ab (b-a) (c-b) (c-a) \end{aligned}$$

$$\therefore \frac{1}{6} ab c (b-a) (c-a) (b-a)$$

9.7



$$(1) \quad \vec{BC} = \vec{C} - \vec{B} \quad \vec{BD} = \vec{D} - \vec{B}$$

$$\vec{S}_1 = \frac{1}{2} (\vec{C} - \vec{B}) \times (\vec{D} - \vec{B})$$

$$= \frac{1}{2} (\vec{C} \times \vec{D} - \vec{B} \times \vec{D} - \vec{C} \times \vec{B})$$

(2) (1) と同様様に

$$\vec{S}_2 = \frac{1}{2} (\vec{D} - \vec{A}) \times (\vec{C} - \vec{A}) = \frac{1}{2} (\vec{D} \times \vec{C} - \vec{D} \times \vec{A} - \vec{A} \times \vec{C})$$

$$\vec{S}_3 = \frac{1}{2} (\vec{B} - \vec{A}) \times (\vec{D} - \vec{A}) = \frac{1}{2} (\vec{B} \times \vec{D} - \vec{A} \times \vec{D} - \vec{B} \times \vec{A})$$

$$\vec{S}_4 = \frac{1}{2} (\vec{C} - \vec{A}) \times (\vec{D} - \vec{A}) = \frac{1}{2} (\vec{C} \times \vec{D} - \vec{A} \times \vec{D} - \vec{C} \times \vec{A})$$

$$\therefore \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + \vec{S}_4 = \vec{0}$$

$$9.8 \quad \phi(x, y, z) = \sqrt{x^2 + y^2 + z^2} = r \quad a < r < b$$

$$\text{grad } \phi = \frac{x}{r} \vec{i} + \frac{y}{r} \vec{j} + \frac{z}{r} \vec{k}$$

$$\Delta \phi = \frac{3}{r} - \frac{x^2 + y^2 + z^2}{r^3} = \frac{2}{r}$$

P.84

$$9.9 \quad \nabla = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}, \quad \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}, \quad r = \sqrt{x^2 + y^2 + z^2}$$

$$\nabla r = \frac{x}{r}\vec{i} + \frac{y}{r}\vec{j} + \frac{z}{r}\vec{k}$$

$$\nabla\left(\frac{1}{r}\right) = -\frac{1}{r^3}(x\vec{i} + y\vec{j} + z\vec{k})$$

$$\frac{\partial}{\partial x} r^n = n r^{n-1} \frac{\partial r}{\partial x} = n r^{n-2} x$$

$$\therefore \nabla(r^n) = n r^{n-2} (x\vec{i} + y\vec{j} + z\vec{k})$$

$$9.10 \quad A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$$

$$(1) |A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & -5 & -7 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & 0 & 18 \end{vmatrix} = -18$$

$$\text{Rank } A = 3$$

$$(2) A^{-1} = \frac{1}{18} \begin{pmatrix} -5 & 1 & 7 \\ 1 & 7 & -5 \\ 7 & -5 & 1 \end{pmatrix}$$

$$\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 1 & 0 \\ 3 & 1 & 2 & 0 & 0 & 1 \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -5 & -2 & 1 & 0 \\ \hline 0 & -5 & -7 & -3 & 0 & 1 \\ \hline 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -5 & -2 & 1 & 0 \\ 0 & 0 & 18 & 7 & -5 & 1 \\ \hline 1 & 0 & 0 & -\frac{5}{18} & \frac{1}{18} & \frac{2}{18} \\ 0 & -1 & 0 & \frac{1}{18} & \frac{7}{18} & \frac{5}{18} \\ 0 & 0 & 1 & \frac{7}{18} & -\frac{5}{18} & \frac{1}{18} \end{array}$$

$$(3) A^{-1}\vec{a} = \frac{1}{18} \begin{pmatrix} -5 & 1 & 7 \\ 1 & 7 & -5 \\ 7 & -5 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{18} \begin{pmatrix} -5 \\ 1 \\ 7 \end{pmatrix}$$

$$A^{-1}\vec{b} = \frac{1}{18} \begin{pmatrix} -5 & 1 & 7 \\ 1 & 7 & -5 \\ 7 & -5 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{18} \begin{pmatrix} 1 \\ 7 \\ -5 \end{pmatrix}$$

$$A^{-1}\vec{c} = \frac{1}{18} \begin{pmatrix} -5 & 1 & 7 \\ 1 & 7 & -5 \\ 7 & -5 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{18} \begin{pmatrix} 7 \\ -5 \\ 1 \end{pmatrix}$$

$$\therefore \text{平行六面体の体積} = \frac{1}{18} \begin{vmatrix} -5 & 1 & 7 \\ 1 & 7 & -5 \\ 7 & -5 & 1 \end{vmatrix} = -\frac{1}{18}$$

$$\frac{1}{18}$$

P 84

$$9.11 \quad P(1, 2, 3) \quad Q(2, 3, 1) \quad R(3, 1, 2)$$

$$\vec{u} = \vec{PQ} = (1, 1, -2) \quad \vec{v} = \vec{PR} = (2, -1, -1)$$

$$(1) \quad \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 2 & -1 & -1 \end{vmatrix} = -3\vec{i} - 3\vec{j} - 3\vec{k}$$

$$(2) \quad \angle QPR = \theta \quad \angle \theta < \pi$$

$$\vec{PQ} \cdot \vec{PR} = |\vec{PQ}| |\vec{PR}| \cos \theta$$

$$3 = \sqrt{6} \sqrt{6} \cos \theta \quad \cos \theta = \frac{1}{2} \quad \theta = \frac{\pi}{3}$$

$$(3) \quad \Delta PQR = \frac{1}{2} |\vec{PQ}| |\vec{PR}| \sin \theta = \frac{1}{2} \sqrt{6} \sqrt{6} \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$$

$$(4) \quad \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 18 - (1 + 8 + 27) = -18$$

$$\therefore \frac{|-18|}{6} = 3$$

$$9.12 \quad A(2, 3, 0), \quad B(0, 1, 2) \quad L: x=y=z \quad P(x, x, x)$$

$$(1) \quad PA^2 + PB^2 = (x-2)^2 + (x-3)^2 + x^2 + x^2 + (x-1)^2 + (x-2)^2 \\ = 6x^2 - 16x + 18 = 6\left(x - \frac{4}{3}\right)^2 + \frac{22}{3} \quad \text{最小値 } \frac{22}{3}$$

$$(2) \quad \vec{PA} \cdot \vec{PB} = x(x-2) + (x-3)(x-1) + x(x-2) \\ = 3x^2 - 8x + 3 = 3\left(x - \frac{4}{3}\right)^2 - \frac{7}{3} \quad \text{最小値 } -\frac{7}{3}$$

$$(3) \quad \vec{PA} \times \vec{PB} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x-2 & x-3 & x \\ x & x-1 & x-2 \end{vmatrix} = (-4x+6)\vec{i} + (4x-4)\vec{j} + 2\vec{k}$$

$$|\vec{PA} \times \vec{PB}|^2 = 32x^2 - 80x + 56 = 8(4x^2 - 10x + 7) \\ = 8\left\{ \left(2x - \frac{5}{2}\right)^2 + \frac{3}{4} \right\}$$

$$\therefore \text{最小値は } \sqrt{6}$$

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§ フーリエ級数

フーリエ級数

 $f(x)$ が 周期 2π の 周期関数

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx, \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx, \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

10.1 $f(x) = |x|$, $-\pi \leq x \leq \pi$, $f(x+2n\pi) = f(x)$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |x| dx = \frac{1}{\pi} \int_0^{\pi} x dx = \frac{1}{\pi} \left[\frac{x^2}{2} \right]_0^{\pi} = \frac{\pi}{2}$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} |x| \cos nx dx = \frac{2}{\pi} \int_0^{\pi} x \cos nx dx = \frac{2}{\pi} \left\{ \left[\frac{1}{n} x \sin nx \right]_0^{\pi} - \frac{1}{n} \int_0^{\pi} \sin nx dx \right\} \\ &= \frac{2}{\pi} \left[\frac{1}{n^2} \cos nx \right]_0^{\pi} = \frac{2}{n^2 \pi} ((-1)^n - 1) = \begin{cases} 0 & n=2k \\ -\frac{4}{n^2 \pi} & n=2m-1 \end{cases} \end{aligned}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} |x| \sin nx dx = 0 \quad \sin nx \text{ 奇関数}$$

$$|x| \sim \frac{\pi}{2} - \sum_{n=1}^{\infty} \frac{4}{(2n-1)^2 \pi} \cos (2n-1)x$$

10.2 $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$

$$\begin{aligned} \int_{-\pi}^{\pi} \cos nx \cos mx dx &= \frac{1}{2} \int_{-\pi}^{\pi} \{ \cos (n+m)x + \cos (n-m)x \} dx \\ &= \begin{cases} 0 & n \neq m \\ \pi & n = m \end{cases} \end{aligned}$$

$$\int_{-\pi}^{\pi} \cos nx \sin mx dx = \frac{1}{2} \int_{-\pi}^{\pi} \{ \sin (n+m)x - \sin (n-m)x \} dx = 0$$

$$\begin{aligned} \int_{-\pi}^{\pi} \sin nx \sin mx dx &= \frac{1}{2} \int_{-\pi}^{\pi} \{ \cos (n-m)x - \cos (n+m)x \} dx \\ &= \begin{cases} 0 & n \neq m \\ \pi & n = m \end{cases} \end{aligned}$$

$$\therefore \frac{1}{\pi} \int_{-\pi}^{\pi} \{ f(x) \}^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

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10.2 a) $f(x) = |\sin ax| \quad [-\pi, \pi] \quad a: \text{実数}$

$$|\sin ax| = |\sin |a|x| \quad \therefore a \geq 0 \text{ とする.}$$

i) $a=0$ のとき $a_k = 0, k=0, 1, 2, 3, \dots$
 $b_k = 0, k=0, 1, 2, 3, \dots$

ii) $a=k$ のとき $k: \text{正整数}$

$$a_0 = \frac{k}{\pi} \int_{-\frac{\pi}{k}}^{\frac{\pi}{k}} |\sin kx| dx = \frac{2k}{\pi} \int_0^{\frac{\pi}{k}} \sin kx dx = \frac{2}{\pi} [-\cos kx]_0^{\frac{\pi}{k}} = \frac{4}{\pi}$$

$$\begin{aligned} a_n &= \frac{k}{\pi} \int_{-\frac{\pi}{k}}^{\frac{\pi}{k}} |\sin kx| \cos nx dx = \frac{2k}{\pi} \int_0^{\frac{\pi}{k}} \sin kx \cos nx dx \\ &= \frac{k}{\pi} \int_0^{\frac{\pi}{k}} (\sin(k+n)x - \sin(n-k)x) dx = \frac{k}{\pi} \left[-\frac{1}{n+k} \cos(n+k)x + \frac{1}{n-k} \cos(n-k)x \right]_0^{\frac{\pi}{k}} \\ &= \frac{k}{\pi} \left(\frac{2k}{k^2-n^2} + \frac{2k}{k^2-n^2} \cos \frac{n}{k} \pi \right) = \frac{2k^2}{\pi(k^2-n^2)} (1 + \cos \frac{n}{k} \pi) \end{aligned}$$

$$a_k = 0 \quad b_n = 0$$

iii) $a > 1$ 整数ではないとき

周期 $\frac{2\pi}{a}$

$$a_0 = \frac{a}{\pi} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} |\sin ax| dx = \frac{2a}{\pi} \left[-\frac{1}{a} \cos ax \right]_0^{\frac{\pi}{a}} = \frac{4}{\pi}$$

$$\begin{aligned} a_n &= \frac{a}{\pi} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} |\sin ax| \cos nx dx = \frac{2a}{\pi} \frac{1}{2} \int_0^{\frac{\pi}{a}} (\sin(a+n)x + \sin(a-n)x) dx \\ &= \frac{a}{\pi} \left[\frac{-1}{a+n} \cos(a+n)x - \frac{1}{a-n} \cos(a-n)x \right]_0^{\frac{\pi}{a}} \\ &= \frac{a}{\pi} \left(\frac{2a}{a^2-n^2} + \frac{2a}{a^2-n^2} \cos \frac{n}{a} \pi \right) \\ &= \frac{2a^2}{\pi(a^2-n^2)} (1 + \cos \frac{n}{a} \pi) \end{aligned}$$

$$b_n = 0$$

iv) $0 < a < 1$ のとき

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} |\sin ax| dx = \frac{2}{\pi} \left[-\frac{1}{a} \cos ax \right]_0^{\pi} = \frac{2}{\pi a} (1 - \cos a\pi)$$

$$\begin{aligned}
 a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} |\sin ax| \cos nx \, dx = \frac{2}{\pi} \int_0^{\pi} \sin ax \cos nx \, dx \\
 &= \frac{1}{\pi} \int_0^{\pi} \{\sin(a+n)x + \sin(a-n)x\} \, dx = \frac{1}{\pi} \left[-\frac{1}{a+n} \cos(a+n)x - \frac{1}{a-n} \cos(a-n)x \right]_0^{\pi} \\
 &= \frac{1}{\pi} \left(\frac{2a}{a^2-n^2} + \frac{2a(-1)^{n-1}}{a^2-n^2} \cos a\pi \right) = \frac{2a}{\pi(a^2-n^2)} (1 + (-1)^{n-1} \cos a\pi)
 \end{aligned}$$

$$b_n = 0$$

10.3 $u_{xx} + u_{yy} + u = 0$ $r=0$ z'' $u=2$ $r \rightarrow \infty$ z'' $u \rightarrow 0$

$$x = r \cos \theta, \quad y = r \sin \theta$$

$$u_r = u_x \cos \theta + u_y \sin \theta \quad u_{\theta} = u_x (-r \sin \theta) + u_y r \cos \theta$$

$$u_{rr} = u_{xx} \cos^2 \theta + 2u_{xy} \cos \theta \sin \theta + u_{yy} \sin^2 \theta$$

$$u_{\theta\theta} = u_{xx} r^2 \sin^2 \theta - 2u_{xy} r^2 \cos \theta \sin \theta + u_{yy} r^2 \cos^2 \theta - u_x r \cos \theta - u_y r \sin \theta$$

$$\therefore u_{rr} + \frac{1}{r^2} u_{\theta\theta} = u_{xx} + u_{yy} - \frac{1}{r} u_r$$

$$\therefore u_{rr} + \frac{1}{r^2} u_{\theta\theta} + \frac{1}{r} u_r + u = 0$$

$u(r, \theta) = X(r) \cdot Y(\theta)$ 変数分離法

$$(r^2 X'' + rX' + r^2 X)Y + XY'' = 0$$

$$\frac{1}{X} (r^2 X'' + rX' + r^2 X) = -\frac{Y''}{Y} = \lambda^2 \text{ (定数)} \text{ とおく}$$

$$r^2 X'' + rX' + (r^2 - \lambda^2) X = 0$$

$$Y'' + \lambda^2 Y = 0$$

$$\therefore \frac{1}{2} Y'^2 + \frac{1}{2} \lambda^2 Y^2 = \frac{a^2}{2} \quad Y' = \pm \lambda \sqrt{\left(\frac{a}{\lambda}\right)^2 - Y^2} \quad \lambda \neq 0$$

$$\sin^{-1} \frac{a}{\lambda} Y = \pm (\lambda \theta + \alpha)$$

$$\therefore Y = \frac{a}{\lambda} \sin(\lambda \theta + \alpha) \quad \lambda = 0 \text{ のときは } Y = a\theta + b$$

$$r^2 X'' + rX' - (\lambda^2 - r^2) X = 0$$

$$X = \sum_{n=0}^{\infty} C_n r^{n+k} \quad \text{とおく}$$

$$X' = \sum_{n=0}^{\infty} (n+k) C_n r^{n+k-1}, \quad X'' = \sum_{n=0}^{\infty} (n+k)(n+k-1) C_n r^{n+k-2}$$

$$\sum_{n=0}^{\infty} (n+k)(n+k-1) C_n r^{n+k} + \sum_{n=0}^{\infty} (n+k) C_n r^{n+k} - \sum_{n=0}^{\infty} C_n \lambda^2 r^{n+k} + \sum_{n=0}^{\infty} C_n r^{n+k+2} = 0$$

$$\therefore \{k(k-1) + k - \lambda^2\} C_0 = 0$$

$$\{k(k+1) + k + 1 - \lambda^2\} C_1 = 0$$

$$\{(n+k)(n+k-1) + (n+k) - \lambda^2\} C_n + C_{n-2} = 0$$

$$\therefore k^2 - \lambda^2 = 0, (k+1)^2 - \lambda^2 = 0, \therefore k = \pm \lambda, C_1 = 0.$$

$$C_n = \frac{-1}{(n+k-\lambda)(n+k+\lambda)} C_{n-2}$$

$$k = \lambda \text{ のとき}$$

$$C_n = \frac{-1}{n(n+2\lambda)} C_{n-2}$$

$$\therefore C_{2n-1} = 0, C_{2n} = \frac{-1}{2n(2n+2\lambda)} C_{2n-2} = \frac{(-1)^2}{2^2 n(n+\lambda) 2^2 (n-1)(n+\lambda-1)} C_{2n-4}$$

$$= \frac{(-1)^n}{2^{2n} n! (n+\lambda) (n+\lambda-1) \cdots \lambda} C_0$$

$$C_0 = \frac{1}{r^\lambda} \Gamma(\lambda+1) \text{ とおくと}$$

$$C_{2n} = \frac{(-1)^n}{2^{2n+\lambda} n! \Gamma(n+\lambda+1)}$$

$$X = r^\lambda \sum_{n=0}^{\infty} \frac{(-1)^n r^{2n}}{2^{2n+\lambda} n! \Gamma(n+\lambda+1)}$$

$$\therefore X = J_\lambda(r) \text{ ベッセル (第) 関数}$$

λ が 整数 であるとき

$$X = C_0 J_\lambda(r) + C_1 J_{-\lambda}(r)$$

$$r \rightarrow 0 \text{ のとき } J_{-\lambda}(r) \rightarrow \infty$$

λ が 整数 であるとき

$$Y_\lambda(r) = \frac{2}{\pi} J_\lambda(r) \left(\log \frac{r}{2} + \gamma \right) + \frac{r^\lambda}{\pi} \sum_{m=0}^{\infty} \frac{(-1)^{m-1} (\lambda_m + \lambda_{m+1})}{2^{2m+\lambda} m! (m+\lambda)!} r^{2m}$$

$$- \frac{r^\lambda}{\pi} \sum_{m=0}^{\lambda-1} \frac{(\lambda-m-1)!}{2^{2m+\lambda} m!} r^{2m} \quad (r > 0, \lambda = 0, 1, 2, \dots)$$

$$h_0 = 0, h_1 = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{\lambda}$$

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$$s = \lim_{p \rightarrow \infty} \left(\sum_{k=1}^p \frac{1}{k} - \log p \right)$$

$$X(r) = C_1 J_\lambda(r) + C_2 Y_\lambda(r)$$

$$r \rightarrow 0 \text{ and } \lambda \neq 0 \Rightarrow J_\lambda(r) \rightarrow 0, \quad Y_\lambda(r) \rightarrow \infty$$

$$\therefore \lambda = 0$$

$$r \rightarrow 0 \text{ and } \lambda \neq 0 \Rightarrow \mu \rightarrow 2, \quad r \rightarrow \infty \text{ and } \lambda \neq 0 \Rightarrow \mu \rightarrow 0 \text{ and } y$$

$$\mu = 2 \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(\frac{r^2}{4}\right)^m$$

$$10.4 \quad V_{xx} + V_{yy} = k^{-2} V_{xx}$$

$$(1) \quad \begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$V_r = V_x \cos \theta + V_y \sin \theta$$

$$V_{rr} = V_{xx} \cos^2 \theta + 2V_{xy} \cos \theta \sin \theta + V_{yy} \sin^2 \theta$$

$$V_\theta = -V_x r \sin \theta + V_y r \cos \theta$$

$$V_{\theta\theta} = V_{xx} r^2 \sin^2 \theta - 2V_{xy} r^2 \cos \theta \sin \theta + V_{yy} r^2 \cos^2 \theta - V_x r \cos \theta - V_y r \sin \theta$$

$$\therefore V_{rr} + \frac{1}{r^2} V_{\theta\theta} = V_{xx} + V_{yy} - \frac{1}{r} V_r$$

$$(2) \quad (1) \text{ f } y$$

$$V_{rr} + \frac{1}{r^2} V_{\theta\theta} + \frac{1}{r} V_r = k^{-2} V_{xx}$$

$$10.5 \quad f(x) = |x| \quad (-\pi \leq x \leq \pi)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |x| dx = \frac{1}{\pi} \left[\frac{x^2}{2} \right]_0^{\pi} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} |x| \cos nx dx = \frac{2}{\pi} \int_0^{\pi} x \cos nx dx = \frac{2}{\pi} \left[\left[\frac{x}{n} \sin nx \right]_0^{\pi} - \frac{1}{n} \int_0^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \right]_0^{\pi} = \frac{2}{n^2\pi} (-1 + (-1)^n)$$

$$\therefore a_{2m} = 0 \quad a_{2m-1} = \frac{-4}{(2m-1)^2\pi}$$

$$|x| = \frac{\pi}{2} - \sum_{m=1}^{\infty} \frac{4}{(2m-1)^2\pi} \cos(2m-1)x \quad x = \pi \text{ is } \text{f } x$$

$$\pi = \frac{\pi}{2} + \frac{4}{\pi} \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots + \frac{1}{(2n-1)^2} + \dots \right)$$

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$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots + \frac{1}{(2n-1)^2} + \dots = \frac{\pi^2}{8}$$

$$10.6 \quad f(x) = \frac{a}{\pi} |x| \quad -\pi \leq x \leq \pi$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{a}{\pi} |x| dx = \frac{a}{\pi^2} \int_0^{\pi} x dx = \frac{a}{2}$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{a}{\pi} |x| \cos nx dx = \frac{2a}{\pi^2} \int_0^{\pi} x \cos nx dx \\ &= \frac{2a}{\pi^2} \left\{ \left[\frac{x}{n} \sin nx \right]_0^{\pi} - \int_0^{\pi} \frac{x}{n} \sin nx dx \right\} \\ &= \frac{2a}{\pi^2} \frac{1}{n^2} [\cos nx]_0^{\pi} = \frac{2a}{\pi^2 n^2} (-1 + (-1)^n) \end{aligned}$$

$$a_{2m} = 0 \quad a_{2n-1} = \frac{-4a}{\pi^2 (2n-1)^2}$$

$$f(x) = \frac{a}{2} - \frac{4a}{\pi^2} \left\{ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots + \frac{1}{(2n-1)^2} \cos(2n-1)x \right\}$$

$$10.7 \quad (1) \quad R \cdot i(t) + \frac{1}{C} \int i(t) dt = V(t) \quad V(t) = V_0 (u_a(t) - u_b(t))$$

$$V_0 = V_0 u_{t=0}$$

$$(2) \quad R \cdot I(s) + \frac{I(s)}{Cs} = \frac{V_0}{s} (e^{-as} - e^{-bs})$$

$$I(s) \frac{RCs+1}{C} = \frac{V_0}{s} (e^{-as} - e^{-bs})$$

$$G(s) = \frac{1}{RCs+1}$$

$$I(s) = \frac{CV_0}{RCs+1} (e^{-as} - e^{-bs})$$

$$\mathcal{L}^{-1} \left(\frac{\frac{V_0}{R}}{s + \frac{1}{RC}} \right) = \frac{V_0}{R} e^{-\frac{t}{RC}}$$

$$i(t) = \frac{V_0}{R} \left(e^{-\frac{t-a}{RC}} u_a(t) - e^{-\frac{t-b}{RC}} u_b(t) \right)$$

$$i(t) = \begin{cases} 0 & t < a \\ \frac{V_0}{R} e^{\frac{a}{RC}} e^{-\frac{t}{RC}} & a < t < b \\ \frac{V_0}{R} \left(e^{\frac{a}{RC}} - e^{\frac{b}{RC}} \right) e^{-\frac{t}{RC}} & t > b \end{cases}$$