P.78 # 第9章 応用数学

§1 複素数

1.1 (1) $e^{\frac{\pi}{3}i} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

(2) $\exp(2 - i\frac{3}{2}\pi) = e^2(\cos(-\frac{3}{2}\pi) + i\sin(\frac{-3}{2}\pi)) = e^2 i$

(3) $e^{-i\frac{\pi}{6}} = \cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

(4) $e^{i\frac{\pi}{6}} = \cos(\frac{\pi}{6}) + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

(5) $\sqrt{i} = (e^{\frac{\pi}{2}i})^{\frac{1}{2}} = e^{\frac{\pi}{4}i} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

$= (e^{\frac{5\pi}{2}i})^{\frac{1}{2}} = e^{\frac{5\pi}{4}i} = \cos\frac{5}{4}\pi + i\sin\frac{5}{4}\pi = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

(6) $(1-\sqrt{3}i)^6 = \{2(\frac{1}{2} - \frac{\sqrt{3}}{2}i)\}^6 = (2e^{-\frac{\pi}{3}i})^6$

$= 2^6 e^{-2\pi i} = 64(\cos(-2\pi) + i\sin(-2\pi)) = 64$

1.2 (1) $i + 1 = \sqrt{2}(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}) = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = \sqrt{2}\, e^{i\frac{\pi}{4}}$

(2) $1 + \sqrt{3}i = 2(\frac{1}{2} + \frac{\sqrt{3}}{2}i) = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}) = 2e^{i\frac{\pi}{3}}$

(3) $-3 = 3(-1) = 3(\cos\pi + i\sin\pi) = 3e^{i\pi}$

(4) $\sqrt{3} + i = 2(\frac{\sqrt{3}}{2} + \frac{1}{2}i) = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}) = 2e^{i\frac{\pi}{6}}$

1.3 $x^3 - 5x^2 + ax + b = 0$ 1根 $1+\sqrt{2}i$ 共役複素 $1-\sqrt{2}i$ も根

∴ $(x-1)^2 + 2 = x^2 - 2x + 3$ で割り切れる

∴ $x^3 - 5x^2 + ax + b = (x-\alpha)(x^2 - 2x + 3)$

$-5 = -\alpha - 2 \quad a = 3 + 2\alpha \quad b = -\alpha\cdot 3$

$\alpha = 3 \quad a = 9 \quad b = -9$

1.4 $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

$\cos 3\theta + i\sin 3\theta = (\cos\theta + i\sin\theta)^3 = \cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$

$= \cos^3\theta - 3\cos\theta\sin^2\theta + i(3\cos^2\theta\sin\theta - \sin^3\theta)$

∴ $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta = 4\cos^3\theta - 3\cos\theta$

$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta = 3\sin\theta - 4\sin^3\theta$

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1.5 $Z(S)=\dfrac{1}{S^2+2bS+1}$

(1) $Z(i\omega)=\dfrac{1}{-\omega^2+2b\omega i+1}=\dfrac{1}{1-\omega^2+i2b\omega}=\dfrac{1-\omega^2-2b\omega i}{(1-\omega^2)^2+4b^2\omega^2}$

∴ 実部 $Re(Z)=\dfrac{1-\omega^2}{(1-\omega^2)^2+4b^2\omega^2}$ 虚部 $Im(Z)=\dfrac{-2b\omega}{(1-\omega^2)^2+4b^2\omega^2}$

$|Z|=\dfrac{1}{\sqrt{(1-\omega^2)^2+4b^2\omega^2}}$

(2) $(1-\omega^2)^2+4b^2\omega^2=\omega^4+2(2b^2-1)\omega^2+1$

$=(\omega^2+2b^2-1)^2+4b^2-4b^4$

$2b^2-1\leq 0$ のとき $|Z|$の最大値 $\dfrac{1}{2|b|\sqrt{1-b^2}}$

$2b^2-1>0$ のとき $|Z|$の最大値 1

1.6 (1) $|z-a|+|z+a|=2|b|$ 必要十分条件

$2|b|=|z-a|+|z+a|\geq|z-a-z-a|=2|a|$

∴ $|b|\geq|a|$

(2) $a=re^{i\theta}$ と表せる $|a|=r$, $z_1=ze^{-i\theta}$ とおくと $ae^{-i\theta}=r$

$|z-a|=|z_1-r|$ $|z+a|=|z_1+r|$

$z_1=x+iy$ とするとき

$|z_1-r|+|z_1+r|=\sqrt{(x-r)^2+y^2}+\sqrt{(x+r)^2+y^2}=2|b|$

$\dfrac{1}{\sqrt{(x-r)^2+y^2}+\sqrt{(x+r)^2+y^2}}=\dfrac{1}{2|b|}$

$\sqrt{(x+r)^2+y^2}-\sqrt{(x-r)^2+y^2}=\dfrac{2xr}{|b|}$

$\sqrt{(x+r)^2+y^2}=|b|+\dfrac{xr}{|b|}$

∴ $(x+r)^2+y^2=|b|^2+2xr+\dfrac{x^2r^2}{|b|^2}$

$\left(1-\dfrac{r^2}{|b|^2}\right)x^2+y^2=|b|^2-r^2$ $(r^2=a^2)$

$\dfrac{x^2}{|b|^2}+\dfrac{y^2}{|b|^2-r^2}=1$ ∴ $-a$, aを焦点とする楕円

$|z_1|$は楕円の中心から楕円周上の点への距離

∴ $|z|=|z_1|$ の最大値 $|b|$ 最小値 $\sqrt{|b|^2-|a|^2}$

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1.7　Pを表す複素数を $re^{i\theta}$ とすると　$i=e^{i\frac{\pi}{2}}$

$\therefore re^{i\theta}(e^{i\frac{\pi}{2}})^3 = re^{i(\theta+\frac{3}{2}\pi)} = r(\cos(\theta+\frac{3}{2}\pi)+i\sin(\theta+\frac{3}{2}\pi))$

270° 回転

1.8　$z\bar{z}+2z=2i$　　$z=x+iy$ とおくと

$x^2+y^2+2x+2yi=2i$

$\therefore x^2+y^2+2x=0$　$y=1$　$\therefore x^2+2x+1=0$　$x=-1$

$\therefore z=-1+i$

1.9　$x^4-81i=0$　$x=re^{i\theta}$　$81i=3^4e^{i(2k+\frac{1}{2})\pi}$

$\therefore x^4=81i$ より $r^4e^{i4\theta}=3^4e^{i(2k+\frac{1}{2})\pi}$

$\therefore r=3$,　$4\theta=(2k+\frac{1}{2})\pi$　$\theta=\frac{k}{2}\pi+\frac{1}{8}\pi$　$k=0,1,2,3$

$\cos^2\frac{\pi}{8}=\frac{1}{2}(1+\cos\frac{\pi}{4})=\frac{1}{4}(2+\sqrt{2})$　$\cos\frac{\pi}{8}=\frac{1}{2}\sqrt{2+\sqrt{2}}$

$\sin^2\frac{\pi}{8}=\frac{1}{2}(1-\cos\frac{\pi}{4})=\frac{1}{4}(2-\sqrt{2})$　$\sin\frac{\pi}{8}=\frac{1}{2}\sqrt{2-\sqrt{2}}$

$\therefore x=\pm\frac{3}{2}(\sqrt{2+\sqrt{2}}+i\sqrt{2-\sqrt{2}}),\ \pm\frac{3}{2}(-\sqrt{2-\sqrt{2}}+i\sqrt{2+\sqrt{2}})$

1.10　$w=\frac{1}{z}$　$x^2+y^2=4$　$z=x+iy$　$w=u+iv$

$|w|=\frac{1}{|z|}=\frac{1}{2}$　$\therefore |w|^2=\frac{1}{4}$

$\therefore u^2+v^2=\frac{1}{4}$　原点を中心とし半径 $\frac{1}{2}$ の円に写像される

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§.2 複素関数

2.1 $w=\dfrac{z-i}{z+i}$ $|z|=2$

$w(z+i)=z-i$ $(w-1)z=-i(w+1)$ $z=\dfrac{-i(w+1)}{w-1}$

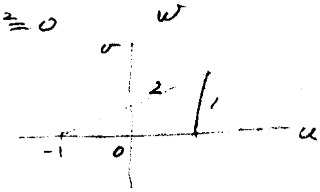
$\therefore \left|\dfrac{w+1}{w-1}\right|=2$ $\therefore |w+1|=2|w-1|$

$w=u+iv$ とおくと $(u+1)^2+v^2=4\{(u-1)^2+v^2\}$

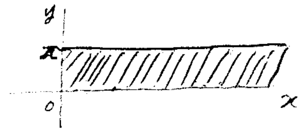
$4(u-1)^2-(u+1)^2+3v^2=0$ $3u^2-10u+3+3v^2=0$

$3(u-\frac{5}{3})^2+3v^2=\frac{25}{3}-3=\frac{16}{3}$

$\therefore (u-\frac{5}{3})^2+v^2=\frac{16}{9}$ アポロニウスの円



2.2 $w=\cosh z$ z 平面上 $x>0,\ 0\leqq y\leqq \pi$



$z=x+iy$ $w=u+iv$ とおく

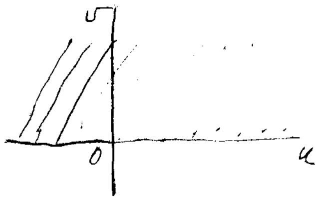
$\cosh z=\frac{1}{2}(e^{x+iy}+e^{-x-iy})$

$=\frac{1}{2}\{e^{x}(\cos y+i\sin y)+e^{-x}(\cos y-i\sin y)\}$

$=\frac{1}{2}(e^{x}+e^{-x})\cos y+\frac{i}{2}(e^{x}-e^{-x})\sin y$

$u=\frac{1}{2}(e^{x}+e^{-x})\cos y$, $v=\frac{1}{2}(e^{x}-e^{-x})\sin y$

$0\leqq y\leqq\pi$ より $v\geqq 0$



2.3 $z=x+iy$, $w=u+iv$ $w=z^2$

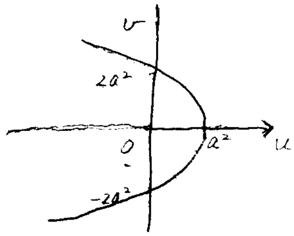
$u+iv=x^2-y^2+2xyi$

$\begin{cases} u=x^2-y^2 \\ v=2xy \end{cases}$ $x=a$ のとき $u=a^2-y^2$ $a\neq 0$ のとき

$v=2ay$ $u=a^2-\frac{v^2}{4a^2}$

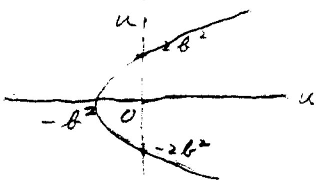
$a=0$ のとき

$u\leqq 0,\ v=0$



$y=b$ のとき $b\neq 0$ のとき $b=0$ のとき

$\begin{cases} u=x^2-b^2 \\ v=2bx \end{cases}$ $u=\dfrac{v^2}{4b^2}-b^2$ $u\geqq 0$, $v=0$



i) $v=0$ $xy=0$

ii) $v=4\sqrt{4-u}$ $v^2=16(4-u)$ $x^2y^2=4(4-x^2+y^2)$

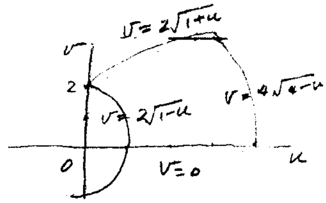
$x^2y^2+4x^2-4y^2=16$ $x^2(y^2+4)-4(y^2+4)=0$

$(x^2-4)(y^2+4)=0$ $\therefore x=2$ $(v\geqq 0)$

iii) $v = 2\sqrt{1+u}$ $\quad v^2 = 4(1+u)$ $\quad x^2y^2 = (1+x^2-y^2)$

$x^2y^2 - x^2 + y^2 - 1 = 0$

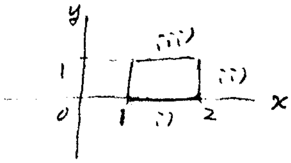
$(x^2+1)(y^2-1) = 0 \quad y = 1,\ (v \geqq 0)$



iv) $v = 2\sqrt{1-u}$ $\quad v^2 = 4(1-u)$ $\quad x^2y^2 = 1-x^2+y^2$

$x^2y^2 + x^2 - y^2 - 1 = 0 \quad (x^2-1)(y^2+1) = 0$

$x = 1$



2. $e^{\pm i\theta} = \cos\theta \pm i\sin\theta$

(1) $e^{i\theta} = \cos\theta + i\sin\theta$ $\qquad \cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$

$e^{-i\theta} = \cos\theta - i\sin\theta$ $\qquad \sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$

$\therefore \tan\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{i(e^{i\theta} + e^{-i\theta})}$

(2) $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$

$\sin\theta = \frac{i}{2i}(e^{i\theta} + e^{-i\theta})$

$\sin^{(n)}\theta = \frac{(i)^n}{2i}(e^{i\theta} - (-1)^n e^{-i\theta})$

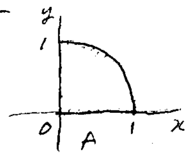
$\sin 0 = 0 \quad \sin' 0 = 1 \qquad n = 2m$ のとき $\sin^{(2m)} 0 = 0$

$n = 2m+1$ のとき $\sin^{(2m+1)}(0) = \frac{(-1)^m}{2} \cdot 2 = (-1)^m$

$\therefore \sin\theta = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots$

$a_{2m} = 0 \quad a_{2m+1} = \frac{(-1)^m}{(2m+1)!}$

2.5

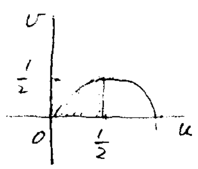


$z = x+iy \qquad x^2+y^2 \leqq 1,\ x \geqq 0,\ y \geqq 0$

$w = \frac{z}{z+1} \qquad w(z+1) = z \quad z(w-1) = -w$

$z = \frac{w}{1-w} = \frac{u+iv}{1-u-iv} = \frac{u(1-u)-v^2+i(uv+v-uv)}{(1-u)^2+v^2}$

$x = \frac{u(1-u)-v^2}{(1-u)^2+v^2},\quad y = \frac{v}{(1-u)^2+v^2}$



$v \geqq 0 \quad u-u^2-v^2 \geqq 0 \quad \frac{1}{4} \geqq (u-\frac{1}{2})^2+v^2$

$\left|\frac{w}{w-1}\right| \leqq 1 \quad |w| \leqq |w-1| \quad u \leqq \frac{1}{2}$

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2.6 $w = f(z) = \sin z \qquad z = x + iy$

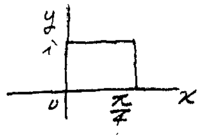
(1) $\sin z = \frac{1}{2i}(e^{iz} - e^{-iz}) = \frac{1}{2i}(e^{ix-y} - e^{-ix+y})$

$= \frac{1}{2i}\{e^{-y}(\cos x + i\sin x) - (\cos x - i\sin x)e^{y}\}$

$= \frac{1}{2i}(e^{-y} - e^{y})\cos x + \frac{1}{2}(e^{-y} + e^{y})\sin x$

$= \frac{i}{2}(e^{y} - e^{-y})\cos x + \frac{1}{2}(e^{y} + e^{-y})\sin x$

$= \sin x \cosh y + i\cos x \sinh y$



(2) $u + iv = \sin x \cosh y + i\cos x \sinh y$

$\therefore u = \sin x \cosh y$

$v = \cos x \sinh y$

$y = 0 \quad v = 0 \qquad 0 \leqq u \leqq \frac{1}{\sqrt{2}}$

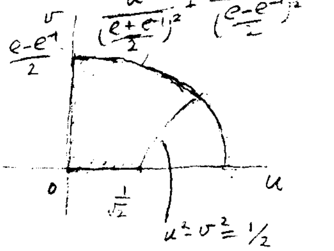
$x = \frac{\pi}{4} \quad u = \frac{1}{\sqrt{2}}\cosh y \quad v = \frac{1}{\sqrt{2}}\sinh y$

$u^2 - v^2 = \frac{1}{2}$

$y = 1 \quad u = \frac{e + e^{-1}}{2}\sin x, \quad v = \frac{e - e^{-1}}{2}\cos x$

$\frac{u^2}{(\frac{e+e^{-1}}{2})^2} + \frac{v^2}{(\frac{e-e^{-1}}{2})^2} = 1$

$x = 0. \quad u = 0 \qquad 0 \leqq v \leqq \frac{e - e^{-1}}{2}$



2.7 $w = \frac{i(1-z)}{1+z} \qquad |z| < 1$

$w(1+z) = i - iz$

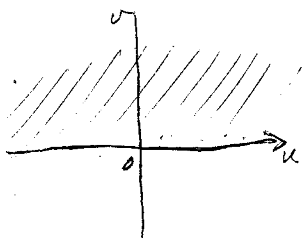
$z(w + i) = i - w$

$z = \frac{i - w}{i + w} \qquad \left|\frac{i-w}{i+w}\right| < 1$

$\therefore |i - w| < |i + w|$

$w = u + iv$ とおくと $u^2 + (v-1)^2 \leqq u^2 + (v+1)^2$

$v \geqq 0$



P.79 § 3 正則関数

3.1 $u=\sin x\cosh y$ $u_x=v_y$ $u_y=-v_x$ $v(0.0)=0$

$v_y=u_x=\cos x\cosh y$

$v_x=-u_y=-\sin x\sinh y$ ∴ $v=\cos x\sinh y+C$ $v(0.0)=0$ より $C=0$

∴ $v(x.y)=\cos x\sinh y$

3.2 $f(z)=u(x.y)+iv(x.y)$ $z=x+iy$ とおくと

$f(z)$ は正則 ∴ $u_x=v_y$ $u_y=-v_x$

$u=v$ のとき $u_x=u_y$ $u_y=-u_x$ ∴ $u_x=u_y=0$

∴ u,v は定数

3.3 $u=-6x^2y+2y^3$ のとき $v(x.y)$ を定めて $w=u+iv$ が正則

∴ $u_x=v_y$. $u_y=-v_x$ ∴ $v_y=-12xy$ $v_x=-(-6x^2+6y^2)=6x^2-6y^2$

∴ $v=-6xy^2+\varphi_1(x)$ $v=2x^3-6xy^2+\varphi_2(y)$ φ_1 は x の関数

∴ $v=2x^3-6xy^2$ φ_2 は y の関数

3.4 $w=(x^2+axy+by^2)+i(cx^2+dxy+y^2)$

$u_x=v_y$ より $2x+ay=dx+2y$ $d=a=2$

$u_y=-v_x$ $ax+2by=-2cx-dy$ $c=-1$ $b=-1$

∴ $w=x^2+2xy-y^2+i(-x^2+2xy+y^2)$

$=x^2+2xyi-y^2-i(x^2+2xyi-y^2)=z^2-iz^2$

∴ $f(z)=z^2-iz^2$

3.5 $f(z)=u+iv$ $z=x+iy$ $u_x=v_y$ $u_y=-v_x$

$u+v=x^3+3x^2y-3xy^2-y^3-x^2+2xy+y^2+1$

$u_x+v_x=u_x-u_y=3x^2+6xy-3y^2-2x+2y$ $2u_x=6x^2-6y^2+4y$

$u_y+v_y=u_y+u_x=3x^2-6xy-3y^2+2x+2y$ $2u_y=-12xy+4x$

$u=x^3-3y^2x+2xy$

$v=3x^2y-y^3-x^2+y^2+1$

$f(z)=z^3-iz^2+i$

P.79 § 4. 複素関数の積分

ローラン展開 $f(z)=\sum_{-\infty}^{\infty} b_n (z-a)^n$ $\quad b_n=\frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)^{n+1}}dz$

4.1 (1) $f(z)=\frac{1}{z^2-(2-i)z-2i}=\frac{1}{(z-2)(z+i)}$ 極 $2, -i$

i) $z=2$ で

$$b_n=\frac{1}{2\pi i}\int_C \frac{1}{(z-2)^{n+2}(z+i)}dz \qquad C: |z-2|=1, \quad z=2+e^{i\theta}, \quad dz=ie^{i\theta}d\theta$$

$$=\frac{1}{2\pi i}\int_0^{2\pi}\frac{ie^{i\theta}}{(2+i+e^{i\theta})e^{(n+2)i\theta}}d\theta$$

$$=\frac{1}{2\pi i}\int_0^{2\pi}\frac{ie^{-i(n+1)\theta}}{(2+i+e^{i\theta})}d\theta$$

$$\frac{1}{2+i+e^{i\theta}}=\frac{1}{2+i}\frac{1}{1+\frac{e^{i\theta}}{2+i}}=\frac{1}{2+i}\left\{1-\frac{e^{i\theta}}{2+i}+\left(\frac{e^{i\theta}}{2+i}\right)^2+\cdots+\left(\frac{-e^{i\theta}}{2+i}\right)^k+\cdots\right\}$$

$$\left|\frac{e^{i\theta}}{2+i}\right|=\frac{1}{|2+i|}<1 \quad \therefore 収束$$

$$\int_0^{2\pi}e^{-im\theta}d\theta=0 \quad (m\neq 0) \qquad \int_0^{2\pi}e^{0}d\theta=2\pi$$

$$=\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{2+i}\left(\frac{-1}{2+i}\right)^{n+1}d\theta=\frac{(-1)^{n+1}}{(2+i)^{n+2}} \qquad n\geq -1$$

$$\therefore b_n=\frac{(-1)^{n+1}}{(2+i)^{n+2}}$$

$$f(z)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2+i)^{n+1}}(z-2)^{n-1}$$

ii) $z=-i$ で

$$b_n=\frac{1}{2\pi i}\int_C\frac{1}{(z-2)(z+i)^{n+2}}dz \qquad C: |z+i|=1, \quad z=-i+e^{i\theta}, \quad dz=ie^{i\theta}d\theta$$

$$=\frac{1}{2\pi i}\int_0^{2\pi}\frac{ie^{i\theta}e^{-i(n+2)\theta}}{-(2+i-e^{i\theta})}d\theta$$

$$=\frac{-1}{2\pi}\int_0^{2\pi}\frac{e^{-i(n+1)\theta}}{(2+i)\left(1-\frac{e^{i\theta}}{2+i}\right)}d\theta$$

$$=\frac{-1}{2\pi}\frac{1}{2+i}\left(\frac{1}{2+i}\right)^{n+1}\cdot 2\pi=-\frac{1}{(2+i)^{n+2}} \qquad n\geq -1$$

$$\therefore b_n=-\frac{1}{(2+i)^{n+2}}$$

$$\therefore f(z)=-\sum_{n=0}^{\infty}\frac{1}{(2+i)^{n+1}}(z+i)^{n-1}$$

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(2) $f(z) = \dfrac{z}{(z+1)(z-2)} \qquad 1<|z|<2$

$$= \frac{1}{3}\left(\frac{2}{z-2} + \frac{1}{z+1}\right)$$

$$= \frac{1}{3}\left\{\frac{-1}{1-\frac{z}{2}} + \frac{1}{1+z}\right\} = \frac{1}{3}\left\{\sum_{n=0}^{\infty}(-1)\left(\frac{z}{2}\right)^n + \sum_{n=0}^{\infty}(-z)^n\right\}$$

$$= \frac{1}{3}\sum_{n=0}^{\infty}\left\{(-1)^n - \frac{1}{2^n}\right\} z^n$$

(3) $f(z) = \dfrac{1}{z(z-1)^3} \qquad 0<|z-1|<1$

$z-1=u$ とおくと

$z=u+1$

$$= \frac{1}{u^3(u+1)}$$

$$= \frac{1}{u^3}\sum_{n=0}^{\infty}(-u)^n = \sum_{n=0}^{\infty}(-1)^n u^{n-3} = \sum_{n=0}^{\infty}(-1)^n (z-1)^{n-3}$$

(4) $f(z) = \dfrac{1}{z(z-1)^3} \qquad |z|<1$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \qquad \frac{1}{(1-z)^2} = \sum_{n=1}^{\infty} n z^{n-1} \qquad \frac{2}{(1-z)^3} = \sum_{n=2}^{\infty} n(n-1) z^{n-2}$$

$$\frac{1}{(1-z)^3} = \sum_{n=0}^{\infty}\frac{(n+2)(n+1)}{2} z^n$$

$$\therefore f(z) = -\sum_{n=0}^{\infty}\frac{(n+2)(n+1)}{2} z^{n-1}$$

4.2 1 $\displaystyle\int_C \frac{1}{(z+i)(z^2+1)}dz \qquad C: |z+i|=1$

$$= \int_C \frac{1}{(z+i)^2(z-i)}dz = \int_C \frac{1}{(z+i)^2\{z+i-2i\}}dz$$

$$= \frac{1}{-2i}\int_C \frac{1}{(z+i)^2}\frac{1}{1-\frac{z+i}{2i}}dz = \frac{-1}{2i}\int_C \sum_{n=0}^{\infty}\frac{(z+i)^{n-2}}{(2i)^n}dz \qquad z+i=e^{i\theta}$$

$dz = i e^{i\theta} d\theta$

$$= -\frac{1}{2}\int_0^{2\pi}\sum_{n=0}^{\infty}\frac{1}{(2i)^n} e^{i(n-1)\theta} d\theta = -\frac{1}{2}\,\frac{1}{2i}\,2\pi = \frac{-\pi}{2i} = \frac{\pi}{2}i$$

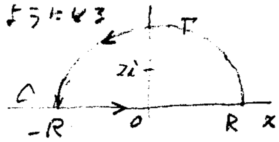
4.3 (1) $\displaystyle\int_0^{\infty}\frac{2x^2-1}{x^4+5x^2+4}dx = \int_0^{\infty}\left(\frac{3}{x^2+4} - \frac{1}{x^2+1}\right)dx$

$$= \left[\frac{3}{2}\tan^{-1}\frac{x}{2} - \tan^{-1}x\right]_0^{\infty} = \frac{3}{2}\cdot\frac{\pi}{2} - \frac{\pi}{2} = \frac{\pi}{4}$$

P.79 ローラン展開 $f(z)=\sum_{-\infty}^{\infty} b_n (z-a)^n$ $\quad b_n=\frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)^{n+1}}dz$

4.3 (2) $\int_{-\infty}^{\infty}\frac{\cos\pi x}{x^2+2x+5} = \int_{-\infty}^{\infty}\frac{\cos \pi x}{(x+1)^2+4}dx$ $\quad x+1=z$ とおくと $x=z-1$ $\quad \cos\pi x=-\cos\pi z$

$\int_{-\infty}^{\infty}\frac{-\cos\pi z}{z^2+4}dz \qquad \int_\Gamma \frac{\cos\pi z}{z^2+4}dz$ $\qquad \Gamma$: 図のようにとる



$$\frac{1}{2\pi i}\int_C \frac{e^{i\pi z}}{z^2+4}dz = Res(2i)$$

$$Res(2i)=\lim_{z\to 2i}(z-2i)\frac{e^{i\pi z}}{z^2+4} = \lim_{z\to 2i}\frac{e^{i\pi z}}{z+2i} = \frac{e^{-2\pi}}{4i}$$

$$\therefore \int_C \frac{e^{i\pi z}}{z^2+4}dz = \frac{\pi e^{-2\pi}}{2}$$

$$\int_C \frac{e^{i\pi z}}{z^2+4}dz = \int_{-R}^{0}\frac{e^{i\pi z}}{z^2+4}dz+\int_0^R \frac{e^{i\pi z}}{z^2+4}dz+\int_\Gamma \frac{e^{i\pi z}}{z^2+4}dz$$

Γ: $z=Re^{i\theta}=R(\cos\theta+i\sin\theta) \qquad 0\le\theta\le\pi$

$$\left|\int_\Gamma \frac{e^{i\pi z}}{z^2+4}dz\right| = \left|\int_0^\pi \frac{e^{i\pi R\cos\theta}\cdot e^{-R\pi\sin\theta}}{R^2e^{i2\theta}+4}\, iRe^{i\theta}d\theta\right|, \quad |e^{iR\pi\cos\theta}|\le 1$$

$$\le \frac{R}{R^2-4}\int_0^\pi e^{-R\pi\sin\theta}d\theta \le \frac{R\pi}{R^2-4}$$

$R\to\infty$ のとき $\int_\Gamma \frac{e^{i\pi z}}{z^2+4}dz \to 0$

$$\int_{-R}^0 \frac{e^{i\pi z}}{z^2+4}dz+\int_0^R \frac{e^{i\pi z}}{z^2+4}dz = \int_0^R \frac{e^{i\pi z}+e^{-i\pi z}}{z^2+4}dz$$

$$=\int_0^R \frac{2\cos\pi z}{z^2+4}dz$$

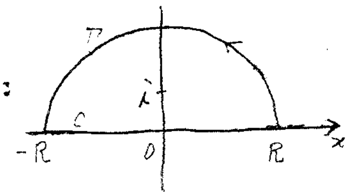
$$2\int_0^\infty \frac{\cos \pi z}{z^2+4}dz = \frac{\pi e^{-2\pi}}{2} \qquad \therefore \int_{-\infty}^\infty \frac{\cos\pi z}{z^2+4}dz = \frac{\pi e^{-2\pi}}{2}$$

$$\therefore \int_{-\infty}^\infty \frac{\cos\pi x}{(x+1)^2+4}dx = \int_{-\infty}^\infty \frac{-\cos\pi z}{z^2+4}dz = -\frac{\pi e^{-2\pi}}{2}$$

4.4 (1) $f(z)=\frac{ze^{iz}}{1+z^2}$ $\quad Res(f, i)=\lim_{z\to i}(z-i)\frac{ze^{iz}}{z^2+1}=\lim_{z\to i}\frac{ze^{iz}}{z+i}$

$$=\frac{ie^{-1}}{2i}=\frac{e^{-1}}{2}$$

(2) $\frac{1}{2\pi i}\int_C \frac{ze^{iz}}{1+z^2}dz = \frac{e^{-1}}{2}$ $\quad C$: 右図のようにとる



$$\therefore \int_C \frac{ze^{iz}}{1+z^2}dz = \pi e^{-1} i$$

P 80

$$\int_C \frac{ze^{iz}}{1+z^2}dz = \int_{-R}^{0} \frac{xe^{ix}}{1+x^2}dz + \int_0^R \frac{xe^{ix}}{1+x^2}dx + \int_\Gamma \frac{ze^{iz}}{1+z^2}dz \qquad \Gamma;\ z=Re^{i\theta}$$

$$\left|\int_\Gamma \frac{ze^{iz}}{1+z^2}dz\right| = \left|\int_0^\pi \frac{Re^{i\theta}e^{iR\cos\theta}\cdot e^{-R\sin\theta}}{1+R^2e^{i2\theta}}\, iRe^{i\theta}d\theta\right|$$

$$= \left|\int_0^\pi \frac{iR^2e^{iR\cos\theta}\cdot e^{-R\sin\theta}}{1+R^2e^{i2\theta}}d\theta\right| \le \frac{R^2}{R^2-1}\int_0^\pi e^{-R\sin\theta}d\theta$$

$$\int_0^\pi e^{-R\sin\theta}d\theta = 2\int_0^{\frac{\pi}{2}} e^{-R\sin\theta}d\theta \le 2\left\{\int_0^\varepsilon d\theta + \int_\varepsilon^{\frac{\pi}{2}} e^{-R\sin\theta}d\theta\right\}$$

$$\le 2\left\{\varepsilon + e^{-R\sin\varepsilon}\left(\frac{\pi}{2}-\varepsilon\right)\right\}$$

$0<\varepsilon<\theta<\frac{\pi}{2}$

$0<\sin\varepsilon<\sin\theta<1$

$0>-\sin\varepsilon>-\sin\theta>-1$

ここで $R\to\infty$ のとき $e^{-R\sin\varepsilon}\to 0$

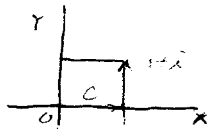
$\therefore \int_0^\pi e^{-R\sin\theta}d\theta \to 0.$ $\quad\therefore \left|\int_\Gamma \frac{ze^{iz}}{1+z^2}dz\right|\to 0$

$$\therefore \int_{-\infty}^0 \frac{xe^{ix}}{1+x^2} + \int_0^\infty \frac{xe^{ix}}{1+x^2}dx = \pi e^{-1}i$$

$$\therefore \int_0^\infty \frac{x(e^{ix}-e^{-ix})}{1+x^2}dx = \pi e^{-1}i \qquad \therefore \int_0^\infty \frac{x\sin x}{1+x^2}dx = \frac{\pi}{2e}$$

$$\therefore \int_{-\infty}^\infty \frac{x\sin x}{1+x^2}dx = \pi e^{-1}$$

4.5 $\quad f(z)=z^2$



$$\int_C f(z)dz = \int_0^1 x^2dx + \int_0^1 (1+iy)^2 i\,dy$$

$$= \left[\frac{x^3}{3}\right]_0^1 + i\left[y+iy^2-\frac{y^3}{3}\right]_0^1$$

$$= \frac{1}{3} + i\left(\frac{2}{3}+i\right) = \frac{-2}{3}+\frac{2}{3}i$$

4.6 (1) $\quad z^4=-1 \qquad z=e^{i\theta}$ とおくと

$e^{i4\theta}=e^{i\pi(2k+1)}$ $\quad\therefore 4\theta=(2k+1)\pi \quad \theta=\frac{k}{2}\pi+\frac{\pi}{4}.\ k=0,1,2,3$

$z=\cos\left(\frac{k}{2}\pi+\frac{\pi}{4}\right)+i\sin\left(\frac{k}{2}\pi+\frac{\pi}{4}\right)$

$k=0,\ z=\frac{1}{\sqrt2}+i\frac{1}{\sqrt2}, \qquad k=1,\ z=-\frac{1}{\sqrt2}+i\frac{1}{\sqrt2}$

$k=2,\ z=-\frac{1}{\sqrt2}-i\frac{1}{\sqrt2}, \qquad k=3,\ z=\frac{1}{\sqrt2}-i\frac{1}{\sqrt2}$

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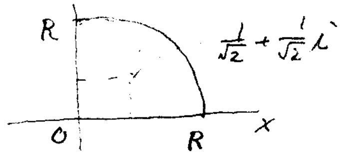
4.6 (2) $\oint_C \frac{1}{z^4+1}\,dz$



$0<R<1$ のとき

$$\oint_C \frac{1}{z^4+1}\,dz = 0$$

$1<R$ のとき

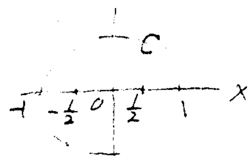
$$\oint_C \frac{1}{z^4+1}\,dz = 2\pi i \operatorname{Res}\left(f, \frac{1}{\sqrt{2}}(1+i)\right)$$

$$\operatorname{Res}\left(f, \frac{1}{\sqrt{2}}(1+i)\right) = \lim_{z\to\frac{1+i}{\sqrt{2}}}\left(z-\frac{1+i}{\sqrt{2}}\right)\frac{1}{\left(z-\frac{1+i}{\sqrt{2}}\right)\left(z+\frac{1+i}{\sqrt{2}}\right)(z^2+i)}$$

$$= \frac{1}{\sqrt{2}(1+i)2i} = \frac{-i(1-i)}{4\sqrt{2}} = \frac{-1-i}{4\sqrt{2}}$$

$$\therefore \oint_C \frac{1}{z^4+1}\,dz = \frac{\pi i(-1-i)}{2\sqrt{2}} = \frac{\pi(1-i)}{2\sqrt{2}} = \frac{\pi(\sqrt{2}-i\sqrt{2})}{4}$$

4.7 $C: |z|=1$



$$\int_C \frac{1}{(2z+1)(2z-1)^3}\,dz$$

$$\therefore \int_C \frac{1}{(2z+1)(2z-1)^3}\,dz = \operatorname{Res}\left(\frac{1}{2}\right)+\operatorname{Res}\left(-\frac{1}{2}\right)$$

$$\operatorname{Res}\left(-\frac{1}{2}\right) = \lim_{z\to-\frac{1}{2}}(2z+1)\frac{1}{(2z+1)(2z-1)^3} = -\frac{1}{8}$$

$$\operatorname{Res}\left(\frac{1}{2}\right) = \frac{1}{2}\lim_{z\to\frac{1}{2}}\frac{d^2}{dz^2}\frac{1}{(2z+1)} = \frac{1}{2}\lim_{z\to\frac{1}{2}}\frac{-2(-4)}{(2z+1)^3} = \frac{1}{2}$$

$$\therefore \int_C \frac{1}{(2z+1)(2z-1)^3}\,dz = 2\pi i\left(-\frac{1}{8}+\frac{1}{2}\right) = \frac{3}{4}\pi i$$

4.8 $f(z) = \frac{z-2}{z^2(z-1)^3}$ 極 0. 2位, 極 1 3位,

$$\operatorname{Res}(f,0) = \lim_{z\to0}\frac{d}{dz}\frac{z-2}{(z-1)^3} = \lim_{z\to0}\frac{z-1-3(z-2)}{(z-1)^4} = \lim_{z\to0}\frac{-2z+5}{(z-1)^4} = 5$$

$$\operatorname{Res}(f,1) = \lim_{z\to1}\frac{1}{2}\frac{d^2}{dz^2}\frac{z-2}{z^2} = \frac{1}{2}\lim_{z\to1}\left(\frac{2}{z^3}-\frac{12}{z^4}\right) = -5$$

P.80

4.9 $f(z)=e^z$ $z=x+iy$ とおく

(1) $f(z)=e^{x+iy}=e^x(\cos y+i\sin y)$

$\therefore \mathrm{Re}(e^z)=e^x\cos y,\ \mathrm{Im}(e^z)=e^x\sin y$

(2) $u=e^x\cos y,\ v=e^x\sin y$

$u_x=e^x\cos y,\ v_y=e^x\cos y \quad \therefore u_x=v_y$

$u_y=-e^x\sin y\ \ v_x=e^x\sin y \quad \therefore u_y=-v_x$

(3) $\int_c f(z)dz=\int_c e^z dz$ $\quad C:\ |z|=1 \ \therefore z=e^{i\theta}$

$f(z)$ は (2) より C の内部で正則である

$\therefore \int_c f(z)dz=0$

4.10 $z=t^2+it^3$ $\quad 0\leqq t\leqq 1$, $i=\sqrt{-1}$

$dz=(2t+i3t^2)dt$

$\therefore \int_c e^z dz=\int_0^1 e^{t^2+it^3}(2t+3t^2i)dt$

$=\int_0^1 e^{t^2}(\cos(t^3)+i\sin(t^3))(2t+i3t^2)dt$

$=\int_0^1 e^{t^2}\{2t\cos(t^3)-3t^2\sin(t^3)\}dt+i\int_0^1 e^{t^2}\{3t^2\cos(t^3)+2t\sin(t^3)\}dt$

$=[e^{t^2}\cos(t^3)]_0^1+\int_0^1 e^{t^2}3t^2\sin(t^3)dt-\int_0^1 e^{t^2}3t^2\sin(t^3)dt$

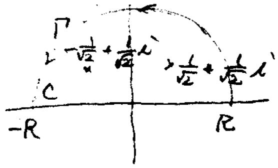
$+i\{[e^{t^2}\sin(t^3)]_0^1-\int_0^1 e^{t^2}3t^2\cos(t^3)dt+\int_0^1 e^{t^2}3t^2\cos t^3 dt$

$=e\cos 1-1+ie\sin 1$

P. 80

4.11 $\int_{-\infty}^{\infty} \frac{1}{x^4+1} dx$ $\quad z^4+1=0$

$(z^2-i)(z^2+i)=0$



$z = \frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}},\ \frac{-1}{\sqrt{2}}+i\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}$

$\frac{1}{2\pi i}\int_C \frac{dz}{z^4+1} = Res\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right) + Res\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)$

$Res\left(\frac{1+i}{\sqrt{2}}\right) = \lim_{z\to\frac{1+i}{\sqrt{2}}} \frac{1}{(z+\frac{1+i}{\sqrt{2}})(z^2+i)} = \frac{1}{\sqrt{2}(1+i)2i} = \frac{-(1-i)i}{4\sqrt{2}} = \frac{-1-i}{4\sqrt{2}}$

$Res\left(\frac{-1+i}{\sqrt{2}}\right) = \lim_{z\to\frac{-1+i}{\sqrt{2}}} \frac{1}{(z^2-i)(z-\frac{1-i}{\sqrt{2}})} = \frac{1}{-2i\sqrt{2}(-1+i)} = \frac{i(-1-i)}{4\sqrt{2}} = \frac{1-i}{4\sqrt{2}}$

$\therefore \int_C \frac{1}{z^4+1} dz = 2\pi i\left(\frac{-1-i+1-i}{4\sqrt{2}}\right) = \frac{\pi}{\sqrt{2}}$

$\int_C \frac{dz}{z^4+1} = \int_{-R}^{0}\frac{1}{x^4+1}dx + \int_0^R \frac{1}{x^4+1}dx + \int_\Gamma \frac{1}{z^4+1}dz$ $\quad \Gamma:\ |z|=R$

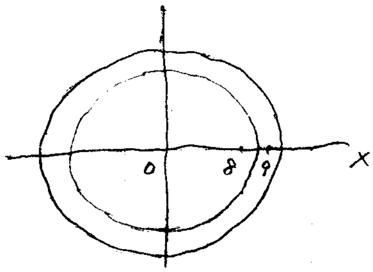
$z=Re^{i\theta}$

$= \int_{-R}^{R}\frac{1}{x^4+1}dx + \int_\Gamma \frac{1}{z^4+1}dz$

$\left|\int_\Gamma \frac{1}{z^4+1}dz\right| = \left|\int_0^\pi \frac{Rie^{i\theta}}{R^4e^{i4\theta}+1}d\theta\right| \le \frac{R}{R^4-1}\int_0^\pi d\theta = \frac{R\pi}{R^4-1}$

$R\to\infty$ のとき $\int_\Gamma \frac{1}{z^4+1}dz \to 0$

$\therefore \int_{-\infty}^{\infty}\frac{1}{x^4+1}dx = \frac{\pi}{\sqrt{2}}$



4.2 $R<8$ のとき

$\int_{C_R}\frac{dz}{(z-8)(z-9)} = 0$

$8<R<9$ のとき

$\int_{C_R}\frac{dz}{(z-8)(z-9)} = 2\pi i\{Res(8)\}$ $\quad Res(8) = \lim_{z\to8}\frac{1}{z-9} = -1$

$\therefore \int_{C_R}\frac{dz}{(z-8)(z-9)} = -2\pi i$

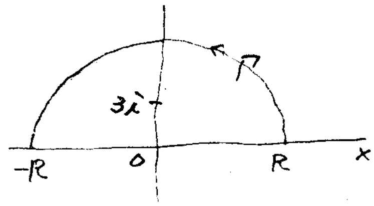
P. 80

4.12 $R > 9$ のとき $\quad \mathrm{Res}(9) = \lim_{z \to 9} \frac{1}{z-8} = 1$

$$\int_{C_R} \frac{1}{(z-8)(z-9)} dz = 2\pi i \left(\mathrm{Res}(8) + \mathrm{Res}(9) \right) = 0$$

4.13

$$\frac{1}{2\pi i} \int_C \frac{e^{ikz}}{z^2+9} dz = \mathrm{Res}(3i)$$



$$\mathrm{Res}(3i) = \lim_{z \to 3i} \frac{e^{ikz}}{z+3i} = \frac{e^{-3k}}{6i}$$

$$\therefore \int_C \frac{e^{ikz}}{z^2+9} dz = \frac{\pi e^{-3k}}{3}$$

$$\int_C \frac{e^{ikz}}{z^2+9} dz = \int_{-R}^{0} \frac{e^{ikx}}{x^2+9} dx + \int_0^R \frac{e^{ikx}}{x^2+9} dx + \int_\Gamma \frac{e^{ikz}}{z^2+9} dz \qquad \Gamma : |z| = R$$

$z = Re^{i\theta}$

$$\left| \int_\Gamma \frac{e^{ikz}}{z^2+9} dz \right| = \left| \int_0^\pi \frac{e^{ikR\cos\theta} \cdot e^{-kR\sin\theta}}{R^2 e^{2i\theta} + 9} Ri e^{i\theta} d\theta \right|$$

$$\leq \frac{R}{R^2-9} \int_0^\pi e^{-kR\sin\theta} d\theta \leq \frac{R\pi}{R^2-9}$$

$R \to \infty$ のとき $\int_\Gamma \frac{e^{ikz}}{z^2+9} dz \to 0$

$$\therefore \int_{-\infty}^{\infty} \frac{e^{ikx}}{x^2+9} dx = \frac{\pi e^{-3k}}{3}$$

P.80

§ 5 確率

5.1 (1) ${}_4C_2\left(\frac{1}{2}\right)^4\cdot{}_3C_1\left(\frac{1}{2}\right)^3\cdot{}_2C_2\left(\frac{1}{2}\right)^2=\frac{4\cdot 3}{2}\cdot\frac{3}{1}\ \frac{1}{2^9}=\frac{9}{2^8}=\frac{9}{256}$

(2) $(a.\ b.\ c)$ $a<c\leqq b\leqq 2$

$b=2,\ c=2,\ a=1,\ 0$ $b=1,\ c=1,\ a=0$

$b=2\ \ c=1.\ a=0$

$(1,\ 2,2)$ のとき ${}_4C_1\left(\frac{1}{2}\right)^4{}_3C_2\left(\frac{1}{2}\right)^3{}_2C_2\left(\frac{1}{2}\right)^2=\frac{3}{128}$

$(0.\ 2,2)$ のとき ${}_4C_0\left(\frac{1}{2}\right)^4{}_3C_2\left(\frac{1}{2}\right)^3{}_2C_2\left(\frac{1}{2}\right)^2=\frac{3}{512}$

$(0,2\ 1)$ のとき ${}_4C_0\left(\frac{1}{2}\right)^4{}_3C_2\left(\frac{1}{2}\right)^3{}_2C_1\left(\frac{1}{2}\right)^2=\frac{3}{256}$

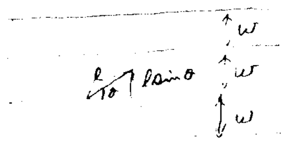
$(0.1\ 1)$ のとき ${}_4C_0\left(\frac{1}{2}\right)^4{}_3C_1\left(\frac{1}{2}\right)^3{}_2C_1\left(\frac{1}{2}\right)^2=\frac{3}{256}$

$\therefore\ \frac{3}{128}+\frac{3}{512}+\frac{3}{256}+\frac{3}{256}=\frac{27}{512}$

(3) $\begin{vmatrix}1&1&1&1\\2&2-a&2&2\\3&3&3+b&3\\4&4&4&4-c\end{vmatrix}=4$ $\begin{vmatrix}1&1&1&1\\0&-a&0&0\\0&0&b&0\\0&0&0&-c\end{vmatrix}=4$ $\therefore\ abc=4$

$\therefore\ (a\ b\ c)=(4\ 1\ 1)\ \ (2.\ 2.\ 1)\ \ (2,1,2)\ \ (1.2.2)$

5.2 問題の $w<l$ を $w>l$ に訂正



針と直線が交差する確率

$\frac{l\sin\theta}{w}$

$\int_0^\pi\frac{l\sin\theta}{w}\frac{d\theta}{\pi}=\frac{l}{w\pi}\left[-\cos\theta\right]_0^\pi=\frac{2l}{w\pi}$

5.3 $m.\ n$ $m^2+2n^2\leqq 72$

(1) $n=5$ のとき $m=1.\ 2.\ 3.\ 4$

$n\leqq 4$ のとき $m=1.\ 2.\ 3.\ 4.\ 5.\ 6$

$n=6$ のとき $\frac{1}{6}$

$n=5,\ m=5$ $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$

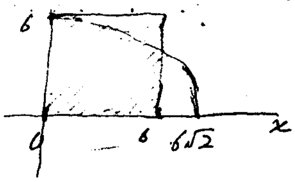
$n=5\ \ m=6$ $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$

$1-\left(\frac{1}{6}+\frac{1}{36}+\frac{1}{36}\right)=1-\frac{2}{9}=\frac{7}{9}$

P.81

5.3 (2) $0 \leqq x \leqq 6,\ 0 \leqq y \leqq 6$ $\quad x^2+2y^2 \leqq 72$

$$\frac{x^2}{72}+\frac{y^2}{36}\leqq 1$$



$$y=\frac{1}{\sqrt{2}}\sqrt{72-x^2}$$

$$=6\sqrt{1-\frac{x^2}{72}} \qquad \frac{x}{\sqrt{72}}=\sin t$$

$$\int_0^6 6\sqrt{1-\frac{x^2}{72}}\,dx = \int_0^{\frac{\pi}{4}} 6\cdot\cos t\,\sqrt{72}\cos t\,dt = 36\sqrt{2}\int_0^{\frac{\pi}{4}}\frac{1+\cos 2t}{2}\,dt$$

$$=18\sqrt{2}\left[t+\frac{1}{2}\sin 2t\right]_0^{\frac{\pi}{4}} = 18\sqrt{2}\left(\frac{\pi}{4}+\frac{1}{2}\right)=\frac{9\sqrt{2}}{2}(\pi+2)$$

$$\therefore\ \frac{1}{36}\cdot\frac{9\sqrt{2}}{2}(\pi+2)=\frac{\sqrt{2}}{8}(\pi+2)$$

5.4

1	0	
0.6	0.4	送信
0.9	0.95	受信

(1) $0.6\times0.9+0.4\times0.05=0.56$

(2) $$\frac{0.6\times0.9}{0.6\times0.9+0.4\times0.05}=\frac{27}{28}$$

5.5 問題でAの袋 青4個とする

	白	赤	青
A	8	3	4
B	4	7	4
C	7	3	5

白→A, 赤→B, 青→C

(1)

1	2	3	
A →	A →	A	$\left(\frac{8}{15}\right)^3$
A →	B →	A	$\frac{3}{15}\cdot\frac{4}{15}$
A →	C →	A	$\frac{4}{15}\cdot\frac{7}{15}$

$$P_{A_3}=\frac{64+12+28}{15^2}=\frac{104}{225}$$

(2)
$$\begin{pmatrix} P_{A_n} \\ P_{B_n} \\ P_{C_n}\end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i\end{pmatrix}^{n-1}\begin{pmatrix} P_{A_1} \\ P_{B_1} \\ P_{C_1}\end{pmatrix}$$

$$=\begin{pmatrix} \frac{8}{15} & \frac{4}{15} & \frac{7}{15} \\ \frac{3}{15} & \frac{7}{15} & \frac{3}{15} \\ \frac{4}{15} & \frac{4}{15} & \frac{5}{15}\end{pmatrix}\begin{pmatrix} P_{A_1} \\ P_{B_1} \\ P_{C_1}\end{pmatrix}$$

p.81

5.5 (3)

$$\begin{vmatrix} 8-\lambda & 4 & 7 \\ 3 & 7-\lambda & 3 \\ 4 & 4 & 5-\lambda \end{vmatrix} = 0$$

$-\lambda^3+20\lambda^2-79\lambda+60=0$

$\lambda^3-20\lambda^2+79\lambda-60=0$

$(\lambda-1)(\lambda-4)(\lambda-15)=0$

$$\begin{array}{rrrr|l} 1 & -20 & 79 & -60 & 1 \\ & 1 & -19 & 60 & \\ \hline 1 & -19 & 60 & 0 & 4 \\ & 4 & -60 & & \\ \hline 1 & -15 & 0 & & \end{array}$$

$\lambda=1$ の固有ベクトル

$$\begin{pmatrix} 7 & 4 & 7 \\ 3 & 6 & 3 \\ 4 & 4 & 4 \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$\lambda=4$ の固有ベクトル

$$\begin{pmatrix} 4 & 4 & 7 \\ 3 & 3 & 3 \\ 4 & 4 & 1 \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$\lambda=15$ の固有ベクトル

$$\begin{pmatrix} -7 & 4 & 7 \\ 3 & -8 & 3 \\ 4 & 4 & -10 \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 34 \\ 21 \\ 22 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 1 & 34 \\ 0 & -1 & 21 \\ -1 & 0 & 22 \end{pmatrix} \qquad A = \begin{pmatrix} \frac{8}{15} & \frac{4}{15} & \frac{7}{15} \\ \frac{3}{15} & \frac{7}{15} & \frac{3}{15} \\ \frac{4}{15} & \frac{4}{15} & \frac{5}{15} \end{pmatrix}$$ とおくと

$$P^{-1}AP = \begin{pmatrix} \frac{1}{15} & 0 & 0 \\ 0 & \frac{4}{15} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad A^n = P\begin{pmatrix} (\frac{1}{15})^n & 0 & 0 \\ 0 & (\frac{4}{15})^n & 0 \\ 0 & 0 & 1 \end{pmatrix}P^{-1}$$

$n \to \infty$ のとき

$$P\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}P^{-1}$$

P^{-1}

$$\begin{array}{rrr|rrr} 1 & 1 & 34 & 1 & 0 & 0 \\ 0 & -1 & 21 & 0 & 1 & 0 \\ -1 & 0 & 22 & 0 & 0 & 1 \\ \hline 1 & 1 & 34 & 1 & 0 & 0 \\ 0 & -1 & 21 & 0 & 1 & 0 \\ 0 & 1 & 56 & 1 & 0 & 1 \\ \hline 1 & 0 & 55 & 1 & 1 & 0 \\ 0 & -1 & 21 & 0 & 1 & 0 \\ 0 & 0 & 77 & 1 & 1 & 1 \\ \hline 1 & 0 & 0 & \frac{22}{77} & \frac{22}{77} & -\frac{55}{77} \\ 0 & -1 & 0 & -\frac{21}{77} & \frac{56}{77} & -\frac{21}{77} \\ 0 & 0 & 1 & \frac{1}{77} & \frac{1}{77} & \frac{1}{77} \end{array}$$

$$P^{-1} = \begin{pmatrix} \frac{22}{77} & \frac{22}{77} & \frac{-55}{77} \\ \frac{21}{77} & -\frac{56}{77} & \frac{21}{77} \\ \frac{1}{77} & \frac{1}{77} & \frac{1}{77} \end{pmatrix}$$

P.81

$$P\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}P^{-1}=\begin{pmatrix}1&1&34\\0&-1&21\\-1&0&22\end{pmatrix}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}P^{-1}$$

$$=\begin{pmatrix}0&0&34\\0&0&21\\0&0&22\end{pmatrix}\begin{pmatrix}\frac{22}{77}&\frac{22}{77}&-\frac{55}{77}\\ \frac{21}{77}&-\frac{56}{77}&\frac{21}{77}\\ \frac{1}{77}&\frac{1}{77}&\frac{1}{77}\end{pmatrix}=\begin{pmatrix}\frac{34}{77}&\frac{34}{77}&\frac{34}{77}\\ \frac{22}{77}&\frac{22}{77}&\frac{22}{77}\\ \frac{21}{77}&\frac{21}{77}&\frac{21}{77}\end{pmatrix}$$

∴ $\frac{34}{77}$

5.6 白4, 黒1, 3回とも白がでない $(\frac{1}{5})^3=\frac{1}{125}$

∴ $1-\frac{1}{125}=\frac{124}{125}$

5.7 1か6が出る確率 $\frac{1}{3}$

Aが4回 ${}_5C_4(\frac{1}{3})^4(\frac{2}{3})=5\cdot\frac{2}{3^5}=\frac{10}{3^5}$

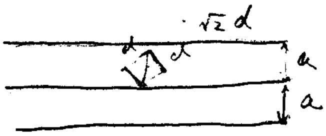
Aが5回 ${}_5C_5(\frac{1}{3})^5=\frac{1}{3^5}$

∴ $\frac{1}{243}+\frac{10}{243}=\frac{11}{243}$

5.8 当たる確率 $\frac{3}{10}$

$\frac{7}{10}\cdot\frac{6}{9}\cdot\frac{3}{8}=\frac{7}{40}$

5.9



$\sqrt{2}d<a$

$\frac{d\sin\theta}{a},\ \frac{d\theta}{\pi}$　　$2\int_0^{\pi}\frac{d\sin\theta}{a}\frac{d\theta}{\pi}=\frac{2d}{a\pi}\left[-\cos\theta\right]_0^{\pi}$

$=\frac{4d}{a\pi}$

5.10 白 2N 赤 1

(1) $P_N={}_NC_1\left(\frac{2N}{2N+1}\right)^{N-1}\frac{1}{2N+1}=\frac{(2N)^N}{2(2N+1)^N}$

(2) $\lim_{N\to\infty}P_N=\lim_{N\to\infty}\frac{1}{2}\frac{1}{(1+\frac{1}{2N})^N}=\frac{1}{2}\lim_{N\to\infty}\frac{1}{\{(1+\frac{1}{2N})^{2N}\}^{\frac{1}{2}}}$

$=\frac{1}{2\sqrt{e}}$

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5.10 (3) $\frac{P_{m+1.N}}{P_{m\,N}} = \frac{{}_NC_{m+1}\frac{(2N)^{N-m-1}}{(2N+1)^N}}{{}_NC_m\frac{(2N)^{N-m}}{(2N+1)^N}} = \frac{\frac{N!}{(m+1)!(N-m-1)!}}{\frac{N!}{m!(N-m)!}} \cdot \frac{1}{2N}$

$= \frac{N-m}{m+1} \cdot \frac{1}{2N} = \frac{N-m}{2N(m+1)}$

(4) $\lim_{N\to\infty} \frac{P_{m+1.N}}{P_{m.N}} = \lim_{N\to\infty} \frac{N-m}{2N(m+1)} = \frac{1}{2(m+1)}$

5.11 (1) $(\frac{1}{2})^3 = \frac{1}{8}$

(2) $1-(\frac{7}{8})^4 = \frac{1695}{4096}$

5.12 赤 3 白 7

$\frac{{}_3C_1 \times {}_7C_1}{{}_{10}C_2} = \frac{3\cdot 7}{\frac{10\cdot 9}{2}} = \frac{7}{15}$

5.13 黒 3 白 3

$\frac{{}_3C_2}{{}_6C_2} = \frac{3\cdot 2}{6\cdot 5} = \frac{1}{5}$

P.82 § 6. 期待値と分散

6.1 20面体 12面赤, 8面青 4回振る.

赤 a 回 青 x 回 $a \geq x$ のとき a 点, $a < x$ のとき 0 点,

$a=4$ のとき $4 \, {}_4C_4 \left(\frac{3}{5}\right)^4 = \frac{324}{5^4}$

$a=3$ のとき $3 \, {}_4C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right) = \frac{648}{5^4}$

$a=2$ のとき $2 \, {}_4C_2 \left(\frac{3}{5}\right)^2 \left(\frac{2}{5}\right)^2 = \frac{432}{5^4}$

$a=3$ のとき最大となりそのときの期待値は $\frac{648}{625}$

6.2 (1) $P(X_1 = x_{1i}) = p_i$, $P(X_2 = x_{2j}) = p_j$ とすると $\sum p_i = 1$, $\sum p_j = 1$,

$$E(X_1 + X_2) = \sum_{j=1}^{n}\sum_{i=1}^{n}(x_{1i} + x_{2j}) p_i p_j = \sum_{j=1}^{n}\sum_{i=1}^{n} x_{1i} p_i p_j + \sum_{j=1}^{n}\sum_{i=1}^{n} x_{2j} p_i p_j$$

$$= \sum_{i=1}^{n} x_{1i} p_i \sum_{j=1}^{n} p_j + \sum_{j=1}^{n} x_{2j} p_j \sum_{i=1}^{n} p_i$$

$$= \sum_{i=1}^{n} x_{1i} p_i + \sum_{j=1}^{n} x_{2j} p_j = E(X_1) + E(X_2)$$

$\therefore E(X_1 + X_2) = E(X_1) + E(X_2)$

(2) $E(X) = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = \frac{7}{2}$

$$E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots + E(X_n)$$

$$= n \cdot \frac{7}{2} = \frac{7}{2} n$$

6.3 (1) A B の二人が勝負する ○: A の勝, ×: A の負

○ × ○

$2 \cdot \left(\frac{1}{2}\right)^2 + 3 \cdot \left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^4 + \cdots + n\left(\frac{1}{2}\right)^n \cdots$

$S_n = 2 \cdot \left(\frac{1}{2}\right)^2 + 3 \cdot \left(\frac{1}{2}\right)^3 + \cdots + (n-1)\left(\frac{1}{2}\right)^{n-1} + n\left(\frac{1}{2}\right)^n$

$\frac{1}{2} S_n = 2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^4 + \cdots + (n-1)\left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^{n+1}$

$\frac{1}{2} S_n = 2 \cdot \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \cdots + \left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^{n+1}$

$= \left(\frac{1}{2}\right)^2 + \frac{\left(\frac{1}{2}\right)^2\left(1 - \left(\frac{1}{2}\right)^{n-1}\right)}{1 - \frac{1}{2}} + n\left(\frac{1}{2}\right)^{n+1}$

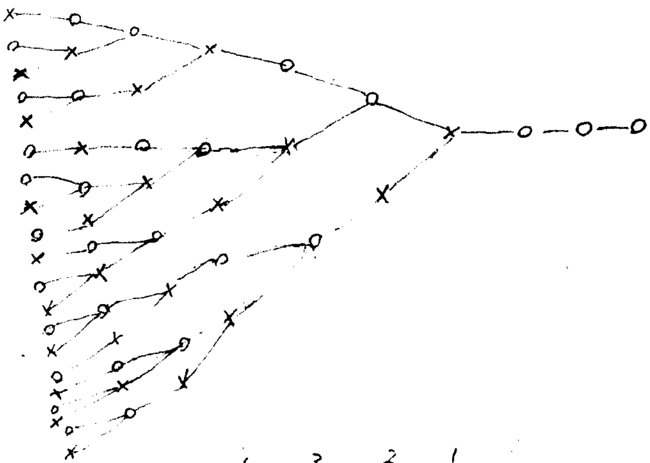
$S_n = \frac{1}{2} + 1 - \left(\frac{1}{2}\right)^{n-1} - n\left(\frac{1}{2}\right)^n$

$\therefore \lim_{n \to \infty} S_n = \frac{3}{2}$

A, B のどちらかが勝つ $2 \times \frac{3}{2} = 3$ 期待値

P. 82

(2) 樹形図



21 13 8 5 3 2 1

$S = 3\cdot\left(\frac{1}{2}\right)^3 + 4\cdot\left(\frac{1}{2}\right)^4 + 5\cdot 2\left(\frac{1}{2}\right)^5 + 6\cdot 3\left(\frac{1}{2}\right)^6 + 7\cdot 5\left(\frac{1}{2}\right)^7 + 8\cdot 8\left(\frac{1}{2}\right)^8 + 9\cdot 13\left(\frac{1}{2}\right)^9$

$+10\cdot 21\left(\frac{1}{2}\right)^{10} + 11\cdot 33\left(\frac{1}{2}\right)^{11} + \cdots$

a_n 1, 1, 2, 3, 5, 8, 13, 21, 33

b_n 0, 1, 1, 2, 3, 5, 8, 12

c_n 0, 1, 1, 2, 3, 4

$b_n = a_{n+1} - a_n \quad c_n = b_{n+1} - b_n$ とおくと

$c_n = n \quad \therefore b_{n+1} - b_1 = \frac{n(n+1)}{2} \quad \therefore b_n = \frac{n^2 - n + 4}{2}$

$a_n - a_1 = \sum_{k=1}^{n-1} \frac{k^2 - k + 4}{2} = \frac{1}{2}\left\{\frac{(n-1)n(2n-1)}{6} - \frac{(n-1)n}{2} + 4(n-1)\right\}$

$= \frac{1}{6}\{(n-1)^3 + 11(n-1)\}$

$\therefore a_n = \frac{1}{6}\{(n-1)^3 + 11(n-1) + 18\}$

$S_n = \frac{1}{2} + 2\cdot\left(\frac{1}{2}\right)^2 + 3\cdot\left(\frac{1}{2}\right)^3 + \cdots + n\left(\frac{1}{2}\right)^n$ とおくと

$\frac{1}{2}S_n = \left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^3 + \cdots + n\left(\frac{1}{2}\right)^{n+1}$

$S_n = 2\left\{\sum_{k=1}^{n}\left(\frac{1}{2}\right)^k - n\left(\frac{1}{2}\right)^{n+1}\right\} = 2\left\{1 - \left(\frac{1}{2}\right)^n - n\left(\frac{1}{2}\right)^{n+1}\right\}$

$= 2 - \left(\frac{1}{2}\right)^{n-1} - n\left(\frac{1}{2}\right)^n$

$\lim S_n = 2$

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$$s_n = \frac{1}{2} + 2^2\left(\frac{1}{2}\right)^2 + 3^2\left(\frac{1}{2}\right)^3 + 4^2\left(\frac{1}{2}\right)^4 + \cdots + n^2\left(\frac{1}{2}\right)^n$$

$$\frac{1}{2}s_n = \left(\frac{1}{2}\right)^2 + 2^2\left(\frac{1}{2}\right)^3 + 3^2\left(\frac{1}{2}\right)^4 + \cdots + n^2\left(\frac{1}{2}\right)^{n+1}$$

$$\frac{1}{2}s_n = \sum_{k=1}^{n}(2k-1)\left(\frac{1}{2}\right)^k - n^2\left(\frac{1}{2}\right)^{n+1} = 2\cdot\left(2-\left(\frac{1}{2}\right)^{n-1} - n\left(\frac{1}{2}\right)^n\right) - 1 + \left(\frac{1}{2}\right)^n - n^2\left(\frac{1}{2}\right)^{n+1}$$

$$= 3 - \left(\frac{1}{2}\right)^{n-2} - n\left(\frac{1}{2}\right)^{n-1} + \left(\frac{1}{2}\right)^n - n^2\left(\frac{1}{2}\right)^{n+1}$$

$$s_n = 6 - \left(\frac{1}{2}\right)^{n-3} - n\left(\frac{1}{2}\right)^{n-2} + \left(\frac{1}{2}\right)^{n-1} - n^2\left(\frac{1}{2}\right)^n$$

$$\therefore \lim_{n\to\infty} s_n = 6$$

同様に

$$t_n = \frac{1}{2} + 2^3\left(\frac{1}{2}\right)^2 + 3^3\left(\frac{1}{2}\right)^3 + 4^3\left(\frac{1}{2}\right)^4 + \cdots + n^3\left(\frac{1}{2}\right)^n$$ とおくと

$$\frac{1}{2}t_n = \left(\frac{1}{2}\right)^2 + 2^3\left(\frac{1}{2}\right)^3 + 3^3\left(\frac{1}{2}\right)^4 + \cdots + n^3\left(\frac{1}{2}\right)^{n+1}$$

$$= \sum(3k^2-3k+1)\left(\frac{1}{2}\right)^k - n^3\left(\frac{1}{2}\right)^{n+1}$$

$$= 3\sum k^2\left(\frac{1}{2}\right)^k - 3\sum k\left(\frac{1}{2}\right)^k + \sum\left(\frac{1}{2}\right)^k - n^3\left(\frac{1}{2}\right)^{n+1}$$

$$t_n = 6\sum k^2\left(\frac{1}{2}\right)^k - 6\sum k\left(\frac{1}{2}\right)^k + 2\sum\left(\frac{1}{2}\right)^k - n^3\left(\frac{1}{2}\right)^n$$

$$\lim_{n\to\infty} t_n = 36 - 12 + 2 = 26$$

$$T_n = \frac{1}{2} + 2^4\left(\frac{1}{2}\right)^2 + 3^4\left(\frac{1}{2}\right)^3 + 4^4\left(\frac{1}{2}\right)^4 + \cdots + n^4\left(\frac{1}{2}\right)^n$$ とおくと

$$T_n = 2\left[\sum 4k^3\left(\frac{1}{2}\right)^k - \sum 6k^2\left(\frac{1}{2}\right)^k + 4\sum k\left(\frac{1}{2}\right)^k - \sum\left(\frac{1}{2}\right)^k\right] - n^4\left(\frac{1}{2}\right)^n$$

$$\lim_{n\to\infty} T_n = 8\times 26 - 12\times 6 + 8\times 2 - 2 = 150$$

$$S = 3\cdot\left(\frac{1}{2}\right)^3 + 4\cdot\left(\frac{1}{2}\right)^4 + 5\cdot 2\left(\frac{1}{2}\right)^5 + 6\cdot 3\left(\frac{1}{2}\right)^6 + 7\cdot 5\left(\frac{1}{2}\right)^7 + 8\cdot 8\left(\frac{1}{2}\right)^8 + 9\cdot 13\left(\frac{1}{2}\right)^9 + \cdots$$

$$= \frac{3}{8} + \frac{1}{4} + \frac{5}{16} + \sum_{k=0}(k+6)\frac{(k^3+11k+18)}{6}\left(\frac{1}{2}\right)^{k+6}$$

$$= \frac{15}{16} + \frac{1}{6}\left(\frac{1}{2}\right)^6\cdot\sum_{k=0}(k^4+6k^3+11k^2+84k+108)\left(\frac{1}{2}\right)^k$$

$$= \frac{15}{16} + \frac{1}{6\times 64}(150 + 6\times 26 + 11\times 6 + 84\times 2 + 108\times 2)$$

$$= \frac{15}{16} + \frac{63}{32} = \frac{93}{32}$$

$\therefore$ 期待値は $\dfrac{93}{16}$

P.82

6.4 (1) $(\frac{1}{2})^3=\frac{1}{8}$

(2) 0 匹 $(\frac{7}{8})^3=\frac{343}{8^3}$ 1 匹 ${}_3C_1(\frac{7}{8})^2\frac{1}{8}=\frac{147}{8^3}$

2 匹 ${}_3C_2\frac{7}{8}(\frac{1}{8})^2=\frac{21}{8^3}$ 3 匹 ${}_3C_3(\frac{1}{8})^3=\frac{1}{8^3}$

平均. $0\times\frac{343}{8^3}+1\times\frac{147}{8^3}+2\times\frac{21}{8^3}+3\times\frac{1}{8^3}=\frac{3}{8}$

分散 $\frac{1}{8^5}(3^2\times343+5^2\times147+13^2\times21+21^2)$

$=\frac{1}{8^5}\times10752=\frac{21}{8^2}=\frac{21}{64}$

(3) ${}_nC_r(\frac{1}{8})^r(\frac{7}{8})^{n-r}$

6.5 (1) $1\times\frac{1}{100}+2\times\frac{99}{100}\cdot\frac{1}{99}+3\times\frac{99}{100}\cdot\frac{98}{99}\frac{1}{98}+\cdots+100\times\frac{1}{100}$

$=\frac{1}{100}(1+2+3+\cdots+100)=\frac{1}{100}\cdot\frac{100\times101}{2}=50.5$ //

(2) 100人目

6.6 (1) メガネ $\frac{2}{5}$. メガネなし $\frac{3}{5}$

$P(0)=(\frac{3}{5})^3=\frac{27}{125}$ $P(1)=3\,\frac{2}{5}\cdot(\frac{3}{5})^2=\frac{54}{125}$

$P(2)=3(\frac{2}{5})^2\frac{3}{5}=\frac{36}{125}$ $P(3)=(\frac{2}{5})^3=\frac{8}{125}$

(2) $E(X)=\frac{1}{125}(0\times27+1\times54+2\times36+3\times8)=\frac{6}{5}$

$V(X)=\frac{1}{125}(6^2\times27+1^2\times54+4^2\times36+9^2\times8)\frac{1}{5^2}$

$=\frac{90}{125}=\frac{18}{25}=0.72$

6.7 赤 3 白 2 ○—○
○ × ×—○

(1) $\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}$ $\frac{2}{5}\times\frac{3}{4}=\frac{3}{10}$ ∴ $\frac{3}{5}$

(2) $E(X)=0\times\frac{2}{5}+1\times\frac{3}{5}=\frac{3}{5}$

$V(X)=(\frac{3}{5})^2\times\frac{2}{5}+(\frac{2}{5})^2\times\frac{3}{5}=\frac{6}{25}$

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§7 確率関数

7.1 $f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$ $\quad N(m, \sigma^2)$

(1) $\frac{x-m}{\sigma} = t$ とおくと $\quad dx = \sigma\, dt$

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}} dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \qquad \frac{t}{\sqrt{2}} = \tau$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} e^{-\tau^2} d\tau = 1$$ ∴ t は $N(0, 1)$ に従う

(2) $N = 500 \quad \bar{x} = 151, \quad s = 15$

$$\therefore f(x) = \frac{1}{15\sqrt{2\pi}} e^{-\frac{(x-151)^2}{2\times 15}} \qquad \frac{x-151}{15} = t$$

$$\int_{145}^{154} f(x)\,dx = \int_{-0.4}^{0.2} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

$$= \int_0^{0.4} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt + \int_0^{0.2} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

$$= 0.1554 + 0.0793 = 0.2347$$

$500 \times 0.2347 = 117.34 \quad \doteqdot \underline{117}$

7.2 $f(x) = \begin{cases} m e^{-mx} & x \geqq 0 \quad (m > 0) \\ 0 & x < 0 \end{cases}$

(1) $F(x) = \int_0^x m e^{-mt} dt = \left[-e^{-mt}\right]_0^x = 1 - e^{-mx}$

$$F(x) = \begin{cases} 1 - e^{-mx} & x \geqq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^{\infty} m x e^{-mx} dx = \left[-x e^{-mx}\right]_0^{\infty} + \int_0^{\infty} e^{-mx} dx$$

$$= \left[-\frac{1}{m} e^{-mx}\right]_0^{\infty} = \frac{1}{m}$$

$$\sigma^2 = \int_{-\infty}^{\infty} \left(x - \frac{1}{m}\right)^2 m e^{-mx} dx = \left[-\left(x-\frac{1}{m}\right)^2 e^{-mx}\right]_0^{\infty} + 2\int_0^{\infty} \left(x - \frac{1}{m}\right) e^{-mx} dx$$

$$= \frac{1}{m^2} - 2\left[\frac{1}{m}\left(x - \frac{1}{m}\right) e^{-mx}\right]_0^{\infty} + 2\,\frac{1}{m}\int_0^{\infty} e^{-mx} dx = \frac{1}{m^2}$$

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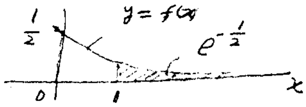
§. 8 統 計

7.2 (3) 平均 2 より $\mu = \frac{1}{m} = 2$ $\therefore m = \frac{1}{2}$ $f(x) = \frac{1}{2} e^{-\frac{1}{2}x}$ $x \geqq 0$

(1) より $F(x) = \begin{cases} 1 - e^{-\frac{1}{2}x} & x > 0 \\ 0 & x < 0 \end{cases}$



$\therefore F(1) = 1 - e^{-\frac{1}{2}}$

平均 $\int_1^\infty (x-1)\frac{1}{2} e^{-\frac{1}{2}x} dx = \left[-(x-1)e^{-\frac{1}{2}x}\right]_1^\infty + \int_1^\infty e^{-\frac{1}{2}x} dx$

$= \left[-2 e^{-\frac{1}{2}x}\right]_1^\infty = 2 e^{-\frac{1}{2}}$

7.3 $P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}$

$\mu = E(X) = \sum k e^{-\lambda}\frac{\lambda^k}{k!} = \lambda \sum e^{-\lambda}\frac{\lambda^{k-1}}{(k-1)!} = \lambda e^{-\lambda} e^{\lambda} = \lambda$

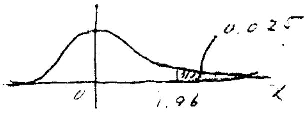
$\sigma^2 = V(X) = \sum (k-\lambda)^2 e^{-\lambda}\frac{\lambda^k}{k!}$

$= \sum e^{-\lambda}\frac{k^2 - 2\lambda k + \lambda^2}{k!}\lambda^k = \lambda^2 \sum e^{-\lambda}\frac{\lambda^k}{k!} - 2\lambda^2 \sum e^{-\lambda}\frac{\lambda^{k-1}}{(k-1)!} + \sum e^{-\lambda}\frac{k^2\lambda^k}{k!}$

$= \lambda^2 - 2\lambda^2 + \sum e^{-\lambda}\frac{k(k-1)+k}{k!}\lambda^k$

$= -\lambda^2 + \lambda^2 \sum e^{-\lambda}\frac{\lambda^{k-2}}{(k-2)!} + \lambda \sum e^{-\lambda}\frac{\lambda^{k-1}}{(k-1)!} = \lambda$

8.1 $\frac{1}{\sqrt{2\pi}}\int_0^{1.96} e^{-\frac{x^2}{2}} dx = 0.475$



$\frac{|\bar{x} - m_0|}{\sigma/\sqrt{n}} \geqq 1.96$ $0.4 \geqq 1.96 \frac{\sigma}{\sqrt{n}}$ $\sigma = 4$

$\sqrt{n} \geqq 1.96 \frac{4}{0.4} = 19.6$ $n \geqq 19.6^2 \fallingdotseq 384.16$

$n \geqq 385$