p. 52

145. (4) 与式 $= \sqrt{\dfrac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}} = \sqrt{\dfrac{2-\sqrt{3}}{2+\sqrt{3}}} = \sqrt{\dfrac{(2-\sqrt{3})^2}{(2+\sqrt{3})(2-\sqrt{3})}} = \sqrt{\dfrac{(2-\sqrt{3})^2}{4-3}} = \sqrt{(2-\sqrt{3})^2} = 2-\sqrt{3}.$

147. $BD = AB = \sqrt{3},\quad AC = 2+\sqrt{3},\quad AD^2 = AC^2 + CD^2 = (2+\sqrt{3})^2 + 1 = 8 + 4\sqrt{3}.$

よって $AD = \sqrt{8+4\sqrt{3}} = 2\sqrt{2+\sqrt{3}}$. よって $\sin 15^\circ = \dfrac{1}{2\sqrt{2+\sqrt{2}}}$,

$\cos 15^\circ = \dfrac{2+\sqrt{3}}{2\sqrt{2+\sqrt{3}}} = \dfrac{(2+\sqrt{3})\sqrt{2+\sqrt{3}}}{2\sqrt{2+\sqrt{3}}\sqrt{2+\sqrt{3}}} = \dfrac{(2+\sqrt{3})\sqrt{2+\sqrt{3}}}{2(2+\sqrt{3})} = \dfrac{\sqrt{2+\sqrt{3}}}{2},$

$\tan 15^\circ = \dfrac{1}{2+\sqrt{3}} = \dfrac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} = \dfrac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

148. 巻末解答参照　条件より $AB = 100(\mathrm{cm}), OH = 90(\mathrm{cm})$ よって $AH = 50\sqrt{2}$,

$\tan\alpha = \dfrac{OH}{AH} = \dfrac{90}{50\sqrt{2}} = \dfrac{9\sqrt{2}}{10} = 1.2728$ よって $\alpha = 52^\circ$. $MH = 50$. よって $\tan\beta = \dfrac{90}{50} = 1.8$ よって $\beta = 61^\circ$.

$OM^2 = OH^2 + MH^2 = 90^2 + 50^2 = 10600. \therefore OM = 10\sqrt{106}$. よって $\tan\dfrac{\gamma}{2} = \dfrac{BM}{OM} = \dfrac{50}{10\sqrt{106}} = 0.4856.$

よって $\dfrac{\gamma}{2} = 26^\circ. \therefore \gamma = 52^\circ$

149. (1) $\cos A = \dfrac{b^2+c^2-a^2}{2bc} = \dfrac{\sqrt{6}^2 + (\sqrt{3}+1)^2 - 2^2}{2\sqrt{6}(\sqrt{3}+1)} = \dfrac{6+3+2\sqrt{3}+1-4}{2\sqrt{6}(\sqrt{3}+1)}$

$= \dfrac{6+2\sqrt{3}}{2\sqrt{6}(\sqrt{3}+1)} = \dfrac{2(3+\sqrt{3})}{2\sqrt{2}(3+\sqrt{3})} = \dfrac{1}{\sqrt{2}}$ よって $A = 45^\circ$

$\cos B = \dfrac{c^2+a^2-b^2}{2ab} = \dfrac{2^2 + (\sqrt{3}+1)^2 - \sqrt{6}^2}{2(\sqrt{3}+1)2} = \dfrac{2+2\sqrt{3}}{4(\sqrt{3}+1)} = \dfrac{2(1+\sqrt{3})}{4(\sqrt{3}+1)} = \dfrac{1}{2}$ よって $B = 60^\circ$

$C = 180^\circ - (A+B) = 180^\circ - (45^\circ + 60^\circ) = 75^\circ$

(2) $\cos A = \dfrac{(3\sqrt{2})^2 + (3-\sqrt{3})^2 - (2\sqrt{3})^2}{2\cdot 3\sqrt{2}(3-\sqrt{3})} = \dfrac{18-6\sqrt{3}}{6\sqrt{2}(3-\sqrt{3})} = \dfrac{6(3-\sqrt{3})}{6\sqrt{2}(3-\sqrt{3})} = \dfrac{1}{\sqrt{2}}$ よって $A = 45^\circ$

$\cos B = \dfrac{(3-\sqrt{3})^2 + (2\sqrt{3})^2 - (3\sqrt{2})^2}{2(3-\sqrt{3})2\sqrt{3}} = \dfrac{6-6\sqrt{3}}{4\sqrt{3}(3-\sqrt{3})} = \dfrac{6(1-\sqrt{3})}{12(\sqrt{3}-1)} = -\dfrac{1}{2}$ よって $B = 120^\circ$

$C = 180^\circ - (A+B) = 180^\circ - (45^\circ + 120^\circ) = 15^\circ$

150. 巻末解答と別の方法：$\cos A = \dfrac{b^2+c^2-a^2}{2bc} = \dfrac{5^2+7^2-8^2}{2\cdot 5\cdot 7} = \dfrac{1}{7}.$

よって $\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\dfrac{1}{7}\right)^2} = \dfrac{4\sqrt{3}}{7}.$

$2R = \dfrac{a}{\sin A} = 8\cdot\dfrac{7}{4\sqrt{3}} = \dfrac{14\sqrt{3}}{3}. \therefore R = \dfrac{7\sqrt{3}}{3}.$

151. 巻末解答参照

153 正弦定理より $\sin A = \dfrac{a}{2R}, \sin B = \dfrac{b}{2R}, \sin C = \dfrac{c}{2R}$. 余弦定理より $\cos A = \dfrac{b^2+c^2-a^2}{2bc}, \cos B = \cdots$ を

(2), (3) は左辺に代入して右辺を導く, (1), (4) は右辺, 左辺 それぞれに代入して別々に整理する.

154. (3) 条件式より $b\tan A = a\tan B, \therefore \dfrac{b\sin A}{\cos A} = \dfrac{a\sin B}{\cos B}$. $\sin A = \dfrac{a}{2R}, \sin B = \dfrac{b}{2R}$ より

$\dfrac{ab}{2R\cos A} = \dfrac{ab}{2R\cos B} \therefore \cos A = \cos B \therefore A = B.$