p.12. 1章§ 2. いろいろな数と式　STEP UP

57. (1) 与式 $= -\dfrac{y}{x}\times\dfrac{y^3}{x^2}\times\left(-\dfrac{x^3}{y^2}\right)=y^2.$　(2) 与式 $=\dfrac{(x+y)(x-y)}{(x-y)^2}\times\left(-\dfrac{x-y}{x(x+y)}\right)=-\dfrac{1}{x}.$

(3) 与式 $=\dfrac{1}{(x-1)(x-3)}-\dfrac{4}{(x+5)(x-3)}+\dfrac{5}{(x+5)(x-1)}=\dfrac{(x+5)-4(x-1)+5(x-3)}{(x-1)(x-3)(x+5)}$

$=\dfrac{2x-6}{(x-1)(x-3)(x+5)}=\dfrac{2(x-3)}{(x-1)(x-3)(x+5)}=\dfrac{2}{(x-1)(x+5)}$

(4) 与式 $=-\dfrac{b+c}{(a-b)(c-a)}-\dfrac{c+a}{(b-c)(a-b)}-\dfrac{a+b}{(c-a)(b-c)}=\dfrac{-(b+c)(b-c)-(c+a)(c-a)-(a+b)(a-b)}{(a-b)(b-c)(c-a)}$

$=\dfrac{-(b^2-c^2)-(c^2-a^2)-(a^2-b^2)}{(a-b)(b-c)(c-a)}=0.$

(5) 与式 $=\dfrac{(2x-y)(x-2y)}{(x-y)^2}\times\dfrac{(x+y)(x-y)}{(3xb-c)(a-b)}-\dfrac{a+b}{(c-a)(b-c)}=\dfrac{-(b+c)(b-c)-(c+a)(c-a)-(a+b)(a-b)}{(a-b)(b-c)(c-a)}$

$=\dfrac{-(b^2-c^2)-(c^2-a^2)-(a^2-b^2)}{(a-b)(b-c)(c-a)}=0.$

58. (1) 与式 $=\dfrac{a-\dfrac{1\times a}{\left(1+\dfrac{1}{a}\right)\times a}}{a+\dfrac{1\times a}{\left(1-\dfrac{1}{a}\right)\times a}}=\dfrac{a-\dfrac{a}{a+1}}{a+\dfrac{a}{a-1}}=\dfrac{\left(a-\dfrac{a}{a+1}\right)\times(a+1)(a-1)}{\left(a+\dfrac{a}{a-1}\right)\times(a+1)(a-1)}=\dfrac{a(a+1)(a-1)-a(a-1)}{a(a+1)(a-1)+a(a+1)}$

$=\dfrac{a(a-1)\{(a+1)-1\}}{a(a+1)\{(a-1)+1\}}=\dfrac{a^2(a-1)}{a^2(a+1)}=\dfrac{a-1}{a+1}.$

(2) 与式 $=1-\dfrac{1}{1-\dfrac{1}{1-\dfrac{1\times x}{\left(1-\dfrac{1}{x}\right)\times x}}}=1-\dfrac{1}{1-\dfrac{1}{1-\dfrac{x}{x-1}}}=1-\dfrac{1}{1-\dfrac{1\times(x-1)}{\left(1-\dfrac{x}{x-1}\right)\times(x-1)}}$

$=1-\dfrac{1}{1-\dfrac{x-1}{(x-1)-x}}=1-\dfrac{1}{1-\dfrac{x-1}{-1}}=1-\dfrac{1}{1+(x-1)}=1-\dfrac{1}{x}=\dfrac{x-1}{x}.$

59. (1) $(x^3+x^2-x+1)\div(x+1)$ の商は x^2-1, 余りは 2. よって $\dfrac{x^3+x^2-x+1}{x+1}=x^2-1+\dfrac{2}{x+1}.$

$(x+x^2-x^3)\div(x-1)$ の商は $-x^2+1$, 余りは 1. よって $\dfrac{x+x^2-x^3}{x-1}=-x^2+1+\dfrac{1}{x-1}.$

与式 $=\left(x^2-1+\dfrac{2}{x+1}\right)+\left(-x^2+1+\dfrac{1}{x-1}\right)=\dfrac{2}{x+1}+\dfrac{1}{x-1}=\dfrac{2(x-1)+(x+1)}{(x+1)(x-1)}=\dfrac{3x-1}{(x+1)(x-1)}.$

(2) $(x^3-2x^2-x+4)\div(x^2-3x+2)$ の商は $x+1$, 余りは 2. よって $\dfrac{x^3-2x^2-x+4}{x^2-3x+2}=x+1+\dfrac{2}{x^2-3x+2}.$

$(x^3-3x^2-x+6)\div(x^2-4x+3)$ の商は $x+1$, 余りは 3. よって $\dfrac{x^3-3x^2-x+6}{x^2-4x+3}=x+1+\dfrac{3}{x^2-4x+3}.$

与式 $=\left(x+1+\dfrac{2}{x^2-3x+2}\right)-\left(x+1+\dfrac{3}{x^2-4x+3}\right)=\dfrac{2}{x^2-3x+2}-\dfrac{3}{x^2-4x+3}$

$=\dfrac{2}{(x-1)(x-2)}-\dfrac{3}{(x-1)(x-3)}=\dfrac{2(x-3)-3(x-2)}{(x-1)(x-2)(x-3)}=-\dfrac{x}{(x-1)(x-2)(x-3)}.$

60. (1) 与式 $=\dfrac{1}{(\sqrt{2}+\sqrt{3})+\sqrt{5}}=\dfrac{(\sqrt{2}+\sqrt{3})-\sqrt{5}}{\{(\sqrt{2}+\sqrt{3})+\sqrt{5}\}\{(\sqrt{2}+\sqrt{3})-\sqrt{5}\}}=\dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2}+\sqrt{3})^2-\sqrt{5}^2}=\dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2+2\sqrt{6}+3-5}$

$=\dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}=\dfrac{\sqrt{12}+\sqrt{18}-\sqrt{30}}{12}=\dfrac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}.$

(2) 与式 $=\dfrac{(1+\sqrt{2})+\sqrt{3}}{\{(1+\sqrt{2})-\sqrt{3}\}\{(1+\sqrt{2})+\sqrt{3}\}}+\dfrac{(1+\sqrt{2})-\sqrt{3}}{\{(1+\sqrt{2})+\sqrt{3}\}\{(1+\sqrt{2})-\sqrt{3}\}}$

$=\dfrac{1+\sqrt{2}+\sqrt{3}}{(1+\sqrt{2})^2-\sqrt{3}^2}+\dfrac{1+\sqrt{2}-\sqrt{3}}{(1+\sqrt{2})^2-\sqrt{3}^2}=\dfrac{1+\sqrt{2}+\sqrt{3}+1+\sqrt{2}-\sqrt{3}}{1+2\sqrt{2}+2-3}=\dfrac{2+2\sqrt{2}}{2\sqrt{2}}=\dfrac{2\sqrt{2}+4}{4}=\dfrac{\sqrt{2}+2}{2}.$

61. (1) $\alpha=a+bi, \beta=c+di$ とおくと $\overline{\alpha+\beta}=\overline{a+bi+c+di}=\overline{(a+c)+(b+d)i}=(a+c)-(b+d)i.$

$\overline{\alpha}+\overline{\beta}=\overline{a+bi}+\overline{c+di}=a-bi+c-di=(a+c)-(b+d)i.$ よって $\overline{\alpha+\beta}=\overline{\alpha}+\overline{\beta}.$

(2) $\alpha=a+bi, \beta=c+di$ とおくと

$|\alpha+\beta|^2=|(a+bi)+(c+di)|=|(a+c)+(b+d)i|^2=\sqrt{(a+c)^2+(b+d)^2}^2=(a+c)^2+(b+d)^2.$

$|\alpha|^2+\alpha\overline{\beta}+\overline{\alpha}\beta+|\beta|^2=|a+bi|^2+(a+bi)\overline{(c+di)}+\overline{(a+bi)}(c+di)+|c+di|^2$

$=\sqrt{a^2+b^2}^2+(a+bi)(c-di)+(a-bi)(c+di)+\sqrt{c^2+d^2}^2$

$=a^2+b^2+ac-adi+bci-bdi^2+ac+adi-bci-bdi^2+c^2+d^2=a^2+b^2+2ac+2bdi^2+c^2+d^2$

$=a^2+2ac+c^2+b^2+2bd+d^2=(a+c)^2+(b+d)^2.$ よって $|\alpha+\beta|^2=|\alpha|^2+\alpha\overline{\beta}+\overline{\alpha}\beta+|\beta|^2$

p.13. PLUS

62. $a>0, b>0$ より $\sqrt{a}, \sqrt{b}$ は実数で $\sqrt{a}>0, \sqrt{b}>0,\ \sqrt{a}^2=a, \sqrt{b}^2=b$ だから

$\sqrt{a+b+2\sqrt{ab}}=\sqrt{\sqrt{a}^2+\sqrt{b}^2+2\sqrt{ab}}=\sqrt{\sqrt{a}^2+2\sqrt{a}\sqrt{b}+\sqrt{b}^2}=\sqrt{(\sqrt{a}+\sqrt{b})^2}=\left|\sqrt{a}+\sqrt{b}\right|=\sqrt{a}+\sqrt{b}.$

63. (1) 与式 $=\sqrt{2+1-2\sqrt{2\cdot1}}=\left|\sqrt{2}-\sqrt{1}\right|=\sqrt{2}-1.$ (2) 与式 $=\sqrt{3+2+2\sqrt{3\cdot2}}=\sqrt{3}+\sqrt{2}.$

(3) 与式 $=\sqrt{7-2\sqrt{12}}\sqrt{4+3-2\sqrt{4\cdot3}}=\left|\sqrt{4}-\sqrt{3}\right|=2-\sqrt{3}.$

(4) 与式 $=\sqrt{27-2\sqrt{50}}\sqrt{25+2-2\sqrt{25\cdot2}}=\left|\sqrt{25}-\sqrt{2}\right|=5-\sqrt{2}.$

(5) 与式 $=\sqrt{\dfrac{4+2\sqrt{3}}{2}}=\dfrac{\sqrt{3+1+2\sqrt{3\cdot1}}}{\sqrt{2}}=\dfrac{\sqrt{3}+\sqrt{1}}{\sqrt{2}}=\dfrac{\sqrt{6}+\sqrt{2}}{2}.$

(6) 与式 $=\sqrt{\dfrac{8+2\sqrt{7}}{2}}=\dfrac{\sqrt{7+1+2\sqrt{7\cdot1}}}{\sqrt{2}}=\dfrac{\sqrt{7}+\sqrt{1}}{\sqrt{2}}=\dfrac{\sqrt{14}+\sqrt{2}}{2}.$

64. (1) $x\geqq1$ より $x>0.$ 与式 $=\sqrt{x+1-2\sqrt{x\cdot1}}=\left|\sqrt{x}-\sqrt{1}\right|.$ $x\geqq1$ より $\sqrt{x}\geqq\sqrt{1}=1.$ よって与式 $=\sqrt{x}-1.$

(2) $a\geqq0$ であり, $a\leqq1$ より $1-a\geqq0.$ また $a+(1-a)=1$ だから

与式 $=\sqrt{a+(1-a)+2\sqrt{a\cdot(1-a)}}=\sqrt{a}+\sqrt{1-a}.$

65. $\sqrt{6-2\sqrt{5}}=\sqrt{5+1-2\sqrt{5\cdot1}}=\left|\sqrt{5}-\sqrt{1}\right|=\sqrt{5}-1.$

$\dfrac{8}{\sqrt{6-2\sqrt{5}}}=\dfrac{8}{\sqrt{5}-1}=\dfrac{8(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)}=\dfrac{8(\sqrt{5}+1)}{5-1}=\dfrac{8(\sqrt{5}+1)}{4}=2\sqrt{5}+2.$

$2\sqrt{5}=\sqrt{20}$ で $4=\sqrt{16}<\sqrt{20}<\sqrt{25}=5$ より $6<2\sqrt{5}+2<7.$

よって $\dfrac{8}{\sqrt{6-2\sqrt{5}}}=2\sqrt{5}+2$ の整数部分は $a=6.$ また $2\sqrt{5}+2=a+b=6+b$ より $b=2\sqrt{5}-4.$

よって $\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{1}{6}+\dfrac{1}{2\sqrt{5}-4}=\dfrac{1}{6}+\dfrac{2\sqrt{5}+4}{(2\sqrt{5}-4)(2\sqrt{5}+4)}=\dfrac{1}{6}+\dfrac{2\sqrt{5}+4}{20-16}=\dfrac{1}{6}+\dfrac{2\sqrt{5}+4}{4}=\dfrac{1}{6}+\dfrac{\sqrt{5}+2}{2}$

$=\dfrac{1+3\sqrt{5}+6}{6}=\dfrac{7+3\sqrt{5}}{6}.$