

p.15. 5章§ 1. 方程式 BASIC

66. 因数分解.

$$(1) (2x+1)(x-3) = 0 \Rightarrow x = -\frac{1}{2}, 3.$$

$$(2) x(3x-2) = 0 \Rightarrow x = 0, \frac{2}{3}.$$

$$(3) (4x+1)(2x+1) = 0 \Rightarrow x = -\frac{1}{4}, -\frac{1}{2}.$$

$$(4) (4x-1)(x-2) = 0 \Rightarrow x = \frac{1}{4}, 2.$$

67. 解の公式.

$$(1) x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-3 \pm \sqrt{5}}{2}.$$

$$(2) x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 3 \cdot (-4)}}{2 \cdot 3} = \frac{-5 \pm \sqrt{73}}{6}.$$

$$(3) x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2} = \frac{4 \pm \sqrt{40}}{4} = \frac{4 \pm 2\sqrt{10}}{4} = \frac{2 \pm \sqrt{10}}{2}.$$

$$(4) x = \frac{-\frac{1}{2} \pm \sqrt{(\frac{1}{2})^2 - 4 \cdot 1 \cdot (-\frac{1}{3})}}{2 \cdot 1} = \frac{-\frac{1}{2} \pm \sqrt{\frac{19}{12}}}{2} = \frac{-\frac{1}{2} \pm \frac{\sqrt{57}}{6}}{2} = \frac{-3 \pm \sqrt{57}}{12}.$$

68. 因数分解 (2重解).

$$(1) (x-5)^2 = 0 \Rightarrow x = 5 (2重解).$$

$$(2) (2x-5)^2 = 0 \Rightarrow x = \frac{5}{2} (2重解).$$

$$(3) \left(x + \frac{2}{3}\right)^2 = 0 \Rightarrow x = -\frac{2}{3} (2重解).$$

$$(4) (3x+1)^2 = 0 \Rightarrow x = -\frac{1}{3} (2重解).$$

69. 解の公式 (虚数解).

$$(1) x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \frac{-1 \pm \sqrt{-23}}{4} = \frac{-1 \pm \sqrt{23}i}{4}.$$

$$(2) x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1} = \frac{5 \pm \sqrt{-3}}{2} = \frac{5 \pm \sqrt{3}i}{2}.$$

$$(3) x^2 = -9 \Leftrightarrow x = \pm\sqrt{-9} = \pm 3i.$$

$$(4) x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2} = 2 \pm i.$$

70. 判別式 $D = b^2 - 4ac$. $D > 0 \Leftrightarrow$ 異なる2つの実数解. $D = 0 \Leftrightarrow$ 2重解. $D < 0 \Leftrightarrow$ 異なる2つの虚数解.

$$(1) D = 3^2 - 4 \cdot 2 \cdot (-4) = 41 > 0 \Leftrightarrow \text{異なる2つの実数解} \quad (2) D = (-2)^2 - 4 \cdot 1 \cdot 5 = -16 < 0 \Leftrightarrow \text{異なる2つの虚数解}$$

$$(3) D = (-1)^2 - 4 \cdot 1 \cdot \frac{1}{4} = 0 \Leftrightarrow 2重解$$

$$(4) D = (-7)^2 - 4 \cdot 3 \cdot 2 = 25 > 0 \Leftrightarrow \text{異なる2つの実数解}$$

71. (1) $D = (3k)^2 - 4 \cdot 1 \cdot 36 = 9k^2 - 144 = 0 \Rightarrow k^2 = 16, k = \pm 4.$

(2) $D = (-2k)^2 - 4 \cdot 1 \cdot (-k+2) = 4k^2 + 4k - 8 = 4(k+2)(k-1) \Rightarrow k = -2, 1.$

(3) $D = \{-2(k+1)\}^2 - 4 \cdot 1 \cdot 4k = 4k^2 - 8k + 4 = 4(k-1)^2 \Rightarrow k = 1.$

72. 解と係数の関係 $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

$$(1) \alpha + \beta = -\frac{4}{3}, \alpha\beta = \frac{-2}{3}. \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(-\frac{4}{3}\right)^2 - 2 \times \left(-\frac{2}{3}\right) = \frac{28}{9}.$$

$$(2) \alpha + \beta = -(-3), \alpha\beta = 5. \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = 3^3 - 3 \times 5 \times 3 = -18.$$

$$(3) \alpha + \beta = -\frac{1}{2}, \alpha\beta = \frac{1}{2}. \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-\frac{1}{2}}{\frac{1}{2}} = -1.$$

73. 因数分解 $ax^2 + bx + c = a(x-\alpha)(x-\beta)$ (α, β は2次方程式 $ax^2 + bx + c = 0$ の解)

$$(1) x^2 - 5x + 5 = 0 \Rightarrow x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{5 \pm \sqrt{5}}{2}. \text{よって } x^2 - 5x + 5 = \left(x - \frac{5 + \sqrt{5}}{2}\right) \left(x - \frac{5 - \sqrt{5}}{2}\right).$$

$$(2) 3x^2 - 7x + 5 = 0 \Rightarrow x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 3 \cdot 5}}{2 \cdot 3} = \frac{7 \pm \sqrt{-11}}{6} = \frac{7 \pm \sqrt{11}i}{6}. \text{よって}$$

$$3x^2 - 7x + 5 = 3 \left(x - \frac{7 + \sqrt{11}i}{6}\right) \left(x - \frac{7 - \sqrt{11}i}{6}\right).$$

$$(3) 2x^2 + 3x - 1 = 0 \Rightarrow x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} = \frac{-3 \pm \sqrt{17}}{4}. \text{よって}$$

$$2x^2 + 3x - 1 = 2 \left(x - \frac{-3 + \sqrt{17}}{4}\right) \left(x - \frac{-3 - \sqrt{17}}{4}\right) = 2 \left(x + \frac{3 - \sqrt{17}}{4}\right) \left(x + \frac{3 + \sqrt{17}}{4}\right).$$

$$(4) x^2 - 2x + 3 = 0 \Rightarrow x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{2 \pm \sqrt{-8}}{2} = \frac{2 \pm 2\sqrt{2}i}{2} = 1 \pm \sqrt{2}i. \text{ よって}$$

$$x^2 - 2x + 3 = \{x - (1 + \sqrt{2}i)\}\{x - (1 - \sqrt{2}i)\} = (x - 1 + \sqrt{2}i)(x - 1 - \sqrt{2}i).$$

$$74. (1) x^2 = X \text{ とおくと } 2X^2 - X - 1 = (2X + 1)(X - 1) = 0 \Rightarrow X = -\frac{1}{2}, 1 \Rightarrow x^2 = -\frac{1}{2}, 1 \Rightarrow x = \pm\frac{\sqrt{2}}{2}i, \pm 1.$$

$$(2) x^3 + 1 = 0. P(x) = x^3 + 1 \text{ とおくと } P(-1) = 0. \text{ 因数定理より } P(x) = (x + 1)(x^2 - x + 1). \text{ よって}$$

$$(x + 1)(x^2 - x + 1) = 0 \Rightarrow x = -1 \text{ または } x^2 - x + 1 = 0. \text{ よって } x = -1, \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} \text{ より}$$

$$x = -1, \frac{1 \pm \sqrt{3}i}{2}.$$

$$(3) P(x) = x^3 - 7x + 6 \text{ とおくと } P(1) = 0. \text{ 因数定理より } P(x) = (x - 1)(x^2 + x - 6) = (x - 1)(x - 2)(x + 3).$$

$$\text{よって } (x - 1)(x - 2)(x + 3) = 0 \Rightarrow x = 1, 2, -3.$$

$$(4) P(x) = x^4 - 4x^3 + 10x^2 - 17x + 10 \text{ とおくと } P(1) = 0. \text{ 因数定理より } P(x) = (x - 1)(x^3 - 3x^2 + 7x - 10).$$

$$Q(x) = x^3 - 3x^2 + 7x - 10 \text{ とおくと } Q(2) = 0 \text{ より } Q(x) = (x - 2)(x^2 - x + 5). \text{ よって}$$

$$P(x) = (x - 1)(x - 2)(x^2 - x + 5) \text{ より } (x - 1)(x - 2)(x^2 - x + 5) = 0 \Rightarrow x = 1, 2 \text{ または } x^2 - x + 5 = 0 \Rightarrow x =$$

$$1, 2, \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$$

$$\text{よって } x = 1, 2, \frac{1 \pm \sqrt{19}i}{2}.$$

$$75. (1) 2x + z = 4 \cdots \textcircled{1}, x + 2y + 3z = 5 \cdots \textcircled{2}, 3x + y + 2z = 6 \cdots \textcircled{3}. \textcircled{1} \text{ より } z = 4 - 2x \cdots \textcircled{4}.$$

$$\textcircled{4} \text{ を } \textcircled{2} \textcircled{3} \text{ に代入して } x + 2y + 3(4 - 2x) = 5, 3x + y + 2(4 - 2x) = 6. \text{ よって } -5x + 2y = -7 \cdots \textcircled{5}, -x + y = -2 \cdots \textcircled{6}.$$

$$\textcircled{6} \times 2 - \textcircled{5} \text{ より } 3x = 3, x = 1. \textcircled{6} \text{ より } -1 + y = -2, y = -1. \textcircled{4} \text{ より } z = 4 - 2 = 2. \text{ よって } x = 1, y = -1, z = 2.$$

$$(2) 3x + 4y - z = 29 \cdots \textcircled{1}, 4x - 2y + 3z = 8 \cdots \textcircled{2}, 2x - 4y - 4z = 6 \cdots \textcircled{3}. \textcircled{1} + \textcircled{3} \text{ より } 5x - 5z = 35, x - z = 7 \cdots \textcircled{4}.$$

$$\textcircled{1} + \textcircled{2} \times 2 \text{ より } 11x + 5z = 45 \cdots \textcircled{5}. \textcircled{4} \times 5 + \textcircled{5} \text{ より } 16x = 80, x = 5. \textcircled{4} \text{ より } 5 - z = 7, z = -2.$$

$$x = 5, z = -2 \text{ を } \textcircled{1} \text{ に代入して } 15 + 4y + 2 = 29, y = 3. \text{ よって } x = 5, y = 3, z = -2.$$

$$76. (1) 2x - y = 1 \cdots \textcircled{1}, x^2 + y^2 + 2y = 4 \cdots \textcircled{2}. \textcircled{1} \text{ より } y = 2x - 1 \cdots \textcircled{3}. \textcircled{2} \text{ に代入して}$$

$$x^2 + (2x - 1)^2 + 2(2x - 1) = 4 \Rightarrow 5x^2 - 5 = 0 \Rightarrow x = \pm 1. \textcircled{3} \text{ より } x = 1 \text{ のとき } y = 1, x = -1 \text{ のとき } y = -3.$$

$$\text{よって } \begin{cases} x = 1 \\ y = 1 \end{cases}, \begin{cases} x = -1 \\ y = -3 \end{cases}.$$

$$(2) x - y = 3 \cdots \textcircled{1}, x^2 - 3xy + y^2 = 7 \cdots \textcircled{2}. \textcircled{1} \text{ より } y = x - 3 \cdots \textcircled{3}. \textcircled{2} \text{ に代入して}$$

$$x^2 - 3x(x - 3) + (x - 3)^2 = 7 \Rightarrow x^2 - 3x - 2 = 0 \Rightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{3 \pm \sqrt{17}}{2}.$$

$$\textcircled{3} \text{ より } y = \frac{3 \pm \sqrt{17}}{2} - 3 = \frac{-3 \pm \sqrt{17}}{2}. \text{ よって } \begin{cases} x = \frac{3 \pm \sqrt{17}}{2} \\ y = \frac{-3 \pm \sqrt{17}}{2} \end{cases} \text{ (復号同順).}$$

$$77. (1) 3x + 2 = \pm 5 \Rightarrow 3x = -2 \pm 5 = 3, -7 \Rightarrow x = 1, -\frac{7}{3}.$$

$$(2) x \geq 0 \text{ より } |x| = x. \text{ よって } 2x = x + 2 \Rightarrow x = 2. \text{ これは } x \geq 0 \text{ をみたすから } x = 2.$$

$$(3) x < 0 \text{ より } |x| = -x. \text{ よって } -2x = x + 2 \Rightarrow x = -\frac{2}{3}. \text{ これは } x < 0 \text{ をみたすから } x = -\frac{2}{3}.$$

$$78. (1) \text{両辺に } 2(x - 1)(x + 2) \text{ をかけると } 2(x + 2) + 2(x - 1) = (x - 1)(x + 2) \text{ より } x^2 - 3x - 4 = (x - 4)(x + 1) = 0.$$

$$\text{よって } x = 4, -1. \text{ これらは無縁解ではない (元の方程式の分母を 0 にしない) ので } x = 4, -1.$$

$$(2) \text{両辺に } (x - 2)(x - 1) \text{ をかけると } x(x - 1) - 4(x - 2) = x + 3 \text{ より } x^2 - 6x + 5 = (x - 1)(x - 5) = 0.$$

$$\text{よって } x = 1, 5. x = 1 \text{ は無縁解 (元の方程式の分母を 0 にする) ので } x = 5.$$

79. (1) 両辺を2乗して $x+3 = (x-3)^2 \Rightarrow x^2 - 7x + 6 = (x-1)(x-6) = 0 \Rightarrow x = 1, 6$.

$x = 1$ のとき $\sqrt{x+3} = \sqrt{4} = 2, x-3 = -2$. よって $x = 1$ は無縁解. 従って $x = 6$.

($x = 6$ のとき $\sqrt{x+3} = \sqrt{9} = 3, x-3 = 3$.)

(2) 両辺を2乗して $x^2 + 16 = (3x-4)^2 \Rightarrow 8x^2 - 24x = 8x(x-3) = 0 \Rightarrow x = 0, 3$.

$x = 0$ のとき $\sqrt{x^2+16} = \sqrt{16} = 4, 3x-4 = -4$. よって $x = 0$ は無縁解. 従って $x = 3$.

($x = 3$ のとき $\sqrt{x^2+16} = \sqrt{25} = 5, 3x-4 = 5$.)

80. (1) 右辺 $= cx^2 + (1-c)x - 1$. 左辺と右辺の係数を比較して $1 = c \cdots \textcircled{1}, a = 1 - c \cdots \textcircled{2}, b = -1 \cdots \textcircled{3}$.

$\textcircled{1}$ より $c = 1$. $\textcircled{3}$ より $b = -1$. $\textcircled{2}$ より $a = 1 - c = 1 - 1 = 0$. よって $a = 0, b = -1, c = 1$.

(2) 右辺 $= ax^2 + bx + (a-2b)$. 左辺と右辺の係数を比較して $2 = a \cdots \textcircled{1}, 3 = b \cdots \textcircled{2}, c = a - 2b \cdots \textcircled{3}$.

$\textcircled{1}$ より $a = 2$. $\textcircled{2}$ より $b = 3$. $\textcircled{3}$ より $c = a - 2b = 2 - 2 \cdot 3 = -4$. よって $a = 2, b = 3, c = -4$.

(3) 右辺 $= x^3 + (c-2)x^2 + (-2c+1)x + c$. 左辺と右辺の係数を比較して $1 = c - 2 \cdots \textcircled{1}, a = -2c + 1 \cdots \textcircled{2}, b = c \cdots \textcircled{3}$.

$\textcircled{1}$ より $c = 3$. $\textcircled{2}$ より $a = -2c + 1 = -2 \cdot 3 + 1 = -5$. $\textcircled{3}$ より $b = c = 3$. よって $a = -5, b = 3, c = 3$.

81. 恒等式 $\Leftrightarrow x$ になにを代入しても成り立つ式.

(1) 両辺に $(x-1)(x-2)$ をかけて $1 = a(x-2) + b(x-1)$. $x = 1$ を代入して $1 = -a$. よって $a = -1$.

$x = 2$ を代入して $1 = b$. よって $b = 1$. 従って $a = -1, b = 1$.

(2) 両辺に $x^3 + 1 = (x+1)(x^2 - x + 1)$ をかけて $7x + 1 = a(x^2 - x + 1) + (bx + c)(x + 1)$.

$x = -1$ を代入して $-6 = 3a$. よって $a = -2$. $x = 0$ を代入して $1 = a + c$. よって $c = 1 - a = 1 - (-2) = 3$.

$x = 1$ を代入して $8 = a + 2b + 2c$. よって $b = \frac{8 - a - 2c}{2} = \frac{8 - (-2) - 2 \cdot 3}{2} = 2$ 従って $a = -2, b = 2, c = 3$.

82. (1) 左辺 $= a^2b - a^2c + b^2c - ab^2 + ac^2 - bc^2 = b^2c - bc^2 + ac^2 - a^2c + a^2b - ab^2$

$= bc(b-c) + ac(c-a) + ab(a-b) =$ 右辺 //

(2) 左辺 $= x^2y^2 + 2xy + 1 + x^2 - 2xy + y^2 = x^2y^2 + x^2 + y^2 + 1 = x^2(y^2 + 1) + y^2 + 1 = (x^2 + 1)(y^2 + 1) =$ 右辺 //

83. $a + b + c = 0$ より $c = -a - b$. よって左辺 $= a^2 - b(-a - b) = a^2 + ab + b^2$, 右辺 $= b^2 - (-a - b)a = b^2 + a^2 + ab$.

従って左辺 = 右辺 //

84. $a : b = c : d$ より $\frac{a}{b} = \frac{c}{d}$. $\frac{a}{b} = \frac{c}{d} = k$ とおけば $a = bk, c = dk$: $\textcircled{1}$

(1) $\textcircled{1}$ より左辺 $= \frac{2bk + 3b}{2bk - 3b} = \frac{b(2k + 3)}{b(2k - 3)} = \frac{2k + 3}{2k - 3}$. 右辺 $= \frac{2dk + 3d}{2dk - 3d} = \frac{d(2k + 3)}{d(2k - 3)} = \frac{2k + 3}{2k - 3}$.

よって左辺 = 右辺 //

(2) $\textcircled{1}$ より左辺 $= \frac{(bk)^2}{b^2} = \frac{b^2k^2}{b^2} = k^2$. 右辺 $= \frac{(bk)^2 - (dk)^2}{b^2 - d^2} = \frac{b^2k^2 - d^2k^2}{b^2 - d^2} = \frac{(b^2 - d^2)k^2}{b^2 - d^2} = k^2$.

よって左辺 = 右辺 //

85. (1) 解の公式. $x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 3 \cdot 3}}{2 \cdot 3} = \frac{-7 \pm \sqrt{13}}{6}$.
- (2) 解の公式. $2x^2 - 3x + 4 = 0$. $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2} = \frac{3 \pm \sqrt{-23}}{4} = \frac{3 \pm \sqrt{23}i}{4}$.
- (3) $(\sqrt{3}x + 1)^2 = 0$. $x = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$ (2重解). (解の公式で解いてもよい)
- (4) $12x^2 - 5x - 2 = (4x + 1)(3x - 2) = 0$. $x = -\frac{1}{4}, \frac{2}{3}$. (解の公式で解いてもよい)
- (5) 解の公式. $3x^2 + 12x - 13 = 0$. $x = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 3 \cdot (-13)}}{2 \cdot 3} = \frac{-12 \pm \sqrt{300}}{6} = \frac{-12 \pm 10\sqrt{3}}{6} = \frac{-6 \pm 5\sqrt{3}}{3}$.
- (6) 解の公式. $6x^2 - 4x + 3 = 0$. $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 6 \cdot 3}}{2 \cdot 6} = \frac{4 \pm \sqrt{-56}}{12} = \frac{4 \pm 2\sqrt{14}i}{12} = \frac{2 \pm \sqrt{14}i}{6}$.
86. (1) $P(x) = x^3 - 4x^2 + 3$ とおくと $P(1) = 0$. 因数定理より $P(x) = (x - 1)(x^2 - 3x - 3)$. よって
 $(x - 1)(x^2 - 3x - 3) = 0 \Rightarrow x = 1$ または $x^2 - 3x - 3 = 0$. よって $x = 1, \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1}$ より
 $x = 1, \frac{3 \pm \sqrt{21}}{2}$.
- (2) $x^2 = X$ とおくと $X^2 - 5X + 4 = (X - 1)(X - 4) = 0 \Rightarrow X = 1, 4 \Rightarrow x^2 = 1, 4 \Rightarrow x = \pm 1, \pm 2$.
- (3) 両辺に $x + 1$ をかけると $x(x + 1) - 4 = 2(x + 1)$ より $x^2 - x - 6 = (x - 3)(x + 2) = 0 \Rightarrow x = 3, -2$.
(分母を 0 にしないので無縁解はない)
- (4) $2x - 5 = \sqrt{x + 8}$ として両辺を 2 乗すると $(2x - 5)^2 = x + 8$. よって $4x^2 - 21x + 17 = (x - 1)(4x - 17) = 0$
 $\Rightarrow x = 1, \frac{17}{4}$. $x = 1$ のとき $2x - \sqrt{x + 8} = 2 - \sqrt{9} = 2 - 3 = -1$. よって $x = 1$ は無縁解. 従って $x = \frac{17}{4}$.
($x = \frac{17}{4}$ のとき $2x - \sqrt{x + 8} = \frac{17}{2} - \sqrt{\frac{17}{4} + 8} = \frac{17}{2} - \frac{7}{2} = 5$.)
- (5) $2x - 3 = \pm 5 \Rightarrow 2x = 3 \pm 5 = 8, -2 \Rightarrow x = 4, -1$.
87. (1) $2x + 3y - z = 9 \cdots \textcircled{1}$, $x + y + z = 2 \cdots \textcircled{2}$, $3x - 2y + 4z = -5 \cdots \textcircled{3}$. $\textcircled{1} + \textcircled{2}$ より $3x + 4y = 11 \cdots \textcircled{4}$.
 $\textcircled{2} \times 4 - \textcircled{3}$ より $x + 6y = 13 \cdots \textcircled{5}$. $\textcircled{5} \times 3 - \textcircled{4}$ より $14y = 28$. よって $y = 2$. $\textcircled{5}$ より $x + 12 = 13$. よって $x = 1$.
 $x = 1, y = 2$ を $\textcircled{2}$ に代入して $1 + 2 + z = 2$ より $z = -1$. よって $x = 1, y = 2, z = -1$.
- (2) $x^2 + 4xy + 2y^2 = 7 \cdots \textcircled{1}$, $x + y = 2 \cdots \textcircled{2}$. $\textcircled{2}$ より $y = 2 - x \cdots \textcircled{3}$. $\textcircled{1}$ に代入して
 $x^2 + 4x(2 - x) + 2(2 - x)^2 = -x^2 + 8 = 7 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$. $\textcircled{3}$ より $x = 1$ のとき $y = 1, x = -1$ のとき $y = 3$.
よって $\begin{cases} x = 1 \\ y = 1 \end{cases}, \begin{cases} x = -1 \\ y = 3 \end{cases}$.
88. $D = (k + 2)^2 - 4 \cdot 1 \cdot (-k + 1) = k^2 + 8k = k(k + 8) = 0$ より $k = 0, -8$.
89. $\alpha + \beta = -\frac{-2}{3} = \frac{2}{3}, \alpha\beta = \frac{4}{3}$
(1) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{2}{3}\right)^2 - 2 \cdot \frac{4}{3} = -\frac{20}{9}$.
(2) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{1}{2}$.
90. (1) $3x^2 - 5x - 1 = 0$ の解は $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 3 \cdot (-1)}}{2 \cdot 3} = \frac{5 \pm \sqrt{37}}{6}$. よって
 $3x^2 - 5x - 1 = 3 \left(x - \frac{5 + \sqrt{37}}{6}\right) \left(x - \frac{5 - \sqrt{37}}{6}\right)$.
- (2) $x^2 + x + 1 = 0$ の解は $x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$. よって
 $x^2 + x + 1 = \left(x - \frac{-1 + \sqrt{3}i}{2}\right) \left(x - \frac{-1 - \sqrt{3}i}{2}\right) = \left(x + \frac{1 - \sqrt{3}i}{2}\right) \left(x + \frac{1 + \sqrt{3}i}{2}\right)$.

91. (1) 右辺 = $a + bx + b + cx^2 + 2cx + c = cx^2 + (b + 2c)x + a + b + c$. 左辺と右辺の係数を比較して $3 = c \cdots \textcircled{1}$,
 $2 = b + 2c \cdots \textcircled{2}$, $1 = a + b + c \cdots \textcircled{3}$. $\textcircled{1}$ より $c = 3$. $\textcircled{2}$ より $b = 2 - 2c = 2 - 6 = -4$. $\textcircled{3}$ より
 $a = 1 - b - c = 1 + 4 - 3 = 2$. よって $a = 2, b = -4, c = 3$.

(別解) $3x^2 + 2x + 1 = (x + 1)\{c(x + 1) + b\} + a$, つまり $P(x) = 3x^3 + 2x + 1$ を $x + 1$ で割ったときの商が $Q(x) = c(x + 1) + b$, 余りが a . さらに $Q(x)$ を $x + 1$ で割ったときの商が c , 余りが b である. よって右のように $P(x)$ を $x + 1$ で割る組立除法を 2 回行って $a = 2, b = -4, c = 3$.

$$\begin{array}{r} -1 \overline{) 3 \quad 2 \quad 1} \\ \underline{-3 \quad 1} \\ -1 \overline{) 3 \quad -1 \quad 2} \\ \underline{-3} \\ 3 \quad -4 \end{array}$$

(2) 両辺に $x^2 + 2x + 1 = (x + 1)^2$ をかけて $3x + 2 = a(x + 1) + b = ax + (a + b)$. 左辺と右辺の係数を比較して
 $3 = a, 2 = a + b$. よって $a = 3, b = -1$.

92. $\frac{a}{x} = \frac{b}{y} = \frac{c}{z} = k$ とおくと $a = kx, b = ky, c = kz$. よって

$$\text{左辺} = \{(kx)^2 + (ky)^2 + (kz)^2\}(x^2 + y^2 + z^2) = (k^2x^2 + k^2y^2 + k^2z^2)(x^2 + y^2 + z^2) = k^2(x^2 + y^2 + z^2)^2.$$

$$\text{右辺} = \{(kx)x + (ky)y + (kz)z\}^2 = (kx^2 + ky^2 + kz^2)^2 = \{k(x^2 + y^2 + z^2)\}^2 = k^2(x^2 + y^2 + z^2)^2.$$

よって左辺 = 右辺.