大日本図書　新基礎数学問題集　5 章§ 3 BASIC 詳しい解答

p.63. 5 章§ 3. 加法定理とその応用　BASIC

319. $\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ = \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{6}+\sqrt{2}}{4}.$

$\cos 105^\circ = \cos(60^\circ + 45^\circ) = \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ = \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}-\sqrt{6}}{4}.$

$\tan 105^\circ = \tan(60^\circ + 45^\circ) = \dfrac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ} = \dfrac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1} = \dfrac{(\sqrt{3}+1)^2}{(1-\sqrt{3})(1+\sqrt{3})} = \dfrac{4+2\sqrt{3}}{1-3} = -2-\sqrt{3}.$

320. $\sin\left(\theta + \dfrac{\pi}{3}\right) = \sin\theta \cos\dfrac{\pi}{3} + \cos\theta \sin\dfrac{\pi}{3} = \dfrac{1}{2}\sin\theta + \dfrac{\sqrt{3}}{2}\cos\theta.$

321. $\cos^2\alpha + \sin^2\alpha = \cos^2\beta + \sin^2\beta = 1$ より $\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(-\dfrac{3}{4}\right)^2 = 1 - \dfrac{9}{16} = \dfrac{7}{16},$

$\sin^2\beta = 1 - \cos^2\beta = 1 - \left(\dfrac{1}{3}\right)^2 = 1 - \dfrac{1}{9} = \dfrac{8}{9}.$ α が第 3 象限の角, β が第 4 象限の角より

$\cos\alpha < 0, \sin\beta < 0$ だから $\cos\alpha = -\dfrac{\sqrt{7}}{4}, \sin\beta = -\dfrac{2\sqrt{2}}{3}.$

(1) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = -\dfrac{3}{4}\cdot\dfrac{1}{3} + \left(-\dfrac{\sqrt{7}}{4}\right)\left(-\dfrac{2\sqrt{2}}{3}\right) = -\dfrac{1}{4} + \dfrac{\sqrt{14}}{6} = \dfrac{-3+2\sqrt{14}}{12}.$

(2) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = -\dfrac{\sqrt{7}}{4}\cdot\dfrac{1}{3} - \left(-\dfrac{3}{4}\right)\left(-\dfrac{2\sqrt{2}}{3}\right) = -\dfrac{\sqrt{7}}{12} - \dfrac{\sqrt{2}}{2} = -\dfrac{\sqrt{7}+6\sqrt{2}}{12}.$

322. $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \dfrac{-2+\frac{1}{5}}{1-(-2)\cdot\frac{1}{5}} = \dfrac{-10+1}{5+2} = -\dfrac{9}{7}.$

323. 2 倍角の公式　$\cos^2\alpha + \sin^2\alpha = 1$ より $\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\dfrac{2}{5}\right)^2 = 1 - \dfrac{4}{25} = \dfrac{21}{25}.$

α が第 2 象限の角より $\cos\alpha < 0$ だから $\cos\alpha = -\dfrac{\sqrt{21}}{5}.$ $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\frac{2}{5}}{-\frac{\sqrt{21}}{5}} = -\dfrac{2}{\sqrt{21}}.$

$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\dfrac{2}{5}\cdot\left(-\dfrac{\sqrt{21}}{5}\right) = -\dfrac{4\sqrt{21}}{25}.$ $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(-\dfrac{\sqrt{21}}{5}\right)^2 - \left(\dfrac{2}{5}\right)^2 = \dfrac{17}{25}.$

$\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2\cdot(-\frac{2}{\sqrt{21}})}{1-(-\frac{2}{\sqrt{21}})^2} = -\dfrac{\frac{4}{\sqrt{21}}}{1-\frac{4}{21}} = -\dfrac{\frac{4}{\sqrt{21}}}{\frac{17}{21}} = -\dfrac{4\cdot 21}{17\sqrt{21}} = -\dfrac{4\cdot 21\sqrt{21}}{17\cdot 21} = -\dfrac{4\sqrt{21}}{17}.$

324. 半角の公式より $\tan^2\dfrac{\frac{\pi}{6}}{2} = \dfrac{1-\cos\frac{\pi}{6}}{1+\cos\frac{\pi}{6}}.$ よって $\tan^2\dfrac{\pi}{12} = \dfrac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}} = \dfrac{2-\sqrt{3}}{2+\sqrt{3}} = \dfrac{(2-\sqrt{3})^2}{(2+\sqrt{3})(2-\sqrt{3})} = \dfrac{(2-\sqrt{3})^2}{4-3},$

$\tan^2\dfrac{\pi}{12} = (2-\sqrt{3})^2.$ $0 < \dfrac{\pi}{12} < \dfrac{\pi}{2}$ より $\tan\dfrac{\pi}{12} > 0$ だから $\tan\dfrac{\pi}{12} = 2-\sqrt{3}.$

325. 半角の公式より $\sin^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{2} = \dfrac{1-\frac{1}{4}}{2} = \dfrac{3}{8}, \cos^2\dfrac{\alpha}{2} = \dfrac{1+\cos\alpha}{2} = \dfrac{1+\frac{1}{4}}{2} = \dfrac{5}{8},$

$\tan^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{1+\cos\alpha} = \dfrac{1-\frac{1}{4}}{1+\frac{1}{4}} = \dfrac{3}{5}.$ $\dfrac{3\pi}{2} < \alpha < 2\pi$ より $\dfrac{3\pi}{4} < \dfrac{\alpha}{2} < \pi.$ よって $\sin\dfrac{\alpha}{2} > 0, \cos\dfrac{\alpha}{2} < 0, \tan\dfrac{\alpha}{2} < 0$

だから $\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{3}{8}} = \dfrac{\sqrt{6}}{4}, \cos\dfrac{\alpha}{2} = -\sqrt{\dfrac{5}{8}} = -\dfrac{\sqrt{10}}{4}, \tan\dfrac{\alpha}{2} = -\sqrt{\dfrac{3}{5}} = -\dfrac{\sqrt{15}}{5}.$

326. 積を和・差に直す公式

(1) 与式 $= \dfrac{1}{2}\{\sin(4\theta+\theta) - \sin(4\theta-\theta)\} = \dfrac{1}{2}(\sin 5\theta - \sin 3\theta).$

(2) 与式 $= -\dfrac{1}{2}\{\cos(3\theta+7\theta) - \cos(3\theta-7\theta)\} = -\dfrac{1}{2}\{\cos 10\theta - \cos(-4\theta)\} = -\dfrac{1}{2}(\cos 10\theta - \cos 4\theta).$

(3) 与式 $= \dfrac{1}{2}\{\cos(5\theta+2\theta) + \cos(5\theta-2\theta)\} = \dfrac{1}{2}(\cos 7\theta + \cos 3\theta).$

(4) 与式 $= \dfrac{1}{2}\{\sin(3\theta+2\theta) + \sin(3\theta-2\theta)\} = \dfrac{1}{2}(\sin 5\theta + \sin\theta).$

327. 和・差を積に直す公式

(1) 与式 $= 2\cos\dfrac{5\theta+3\theta}{2}\sin\dfrac{5\theta-3\theta}{2} = 2\cos 4\theta\sin\theta.$

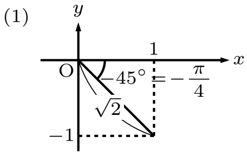
(2) 与式 $= -2\sin\dfrac{2\theta+4\theta}{2}\sin\dfrac{2\theta-4\theta}{2} = -2\sin 3\theta\sin(-\theta) = 2\sin 3\theta\sin\theta.$

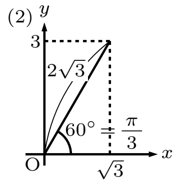
(3) 与式 $= 2\cos\dfrac{\theta+5\theta}{2}\cos\dfrac{\theta-5\theta}{2} = 2\cos 3\theta\cos(-2\theta) = 2\cos 3\theta\cos 2\theta.$

(4) 与式 $= 2\sin\dfrac{\theta+3\theta}{2}\cos\dfrac{\theta-3\theta}{2} = 2\sin 2\theta\cos(-\theta) = 2\sin 2\theta\cos\theta.$

328. 三角関数の合成

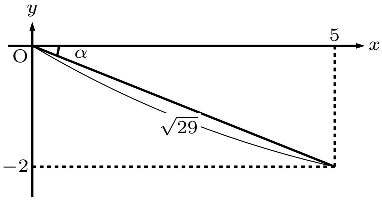
(1) $\sqrt{1^2+(-1)^2} = \sqrt{2}$ より $y = \sqrt{2}\sin\left(x - \dfrac{\pi}{4}\right).$





(2) $\sqrt{\sqrt{3}^2+3^2} = 2\sqrt{3}$ より $y = 2\sqrt{3}\sin\left(x + \dfrac{\pi}{3}\right).$

329.



$\sqrt{5^2+(-2)^2} = \sqrt{29}$. 図より $y = \sqrt{29}\sin(x+\alpha)$. $-1 \leqq \sin(x+\alpha) \leqq 1$ だから y の最大値は $\sqrt{29}$, 最小値は $-\sqrt{29}$.

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330. $\cos^2\alpha + \sin^2\alpha = \cos^2\beta + \sin^2\beta = 1$ より $\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\dfrac{\sqrt{2}}{3}\right)^2 = 1 - \dfrac{2}{9} = \dfrac{7}{9}$,

$\sin^2\beta = 1 - \cos^2\beta = 1 - \left(-\dfrac{2}{5}\right)^2 = 1 - \dfrac{4}{25} = \dfrac{21}{25}$. α, β が第 2 象限の角より

$\cos\alpha < 0, \sin\beta > 0$ だから $\cos\alpha = -\dfrac{\sqrt{7}}{3}, \sin\beta = \dfrac{\sqrt{21}}{5}$.

(1) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \dfrac{\sqrt{2}}{3}\cdot\left(-\dfrac{2}{5}\right) + \left(-\dfrac{\sqrt{7}}{3}\right)\cdot\dfrac{\sqrt{21}}{5} = -\dfrac{2\sqrt{2}}{15} - \dfrac{7\sqrt{3}}{15} = -\dfrac{2\sqrt{2}+7\sqrt{3}}{15}.$

(2) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = -\dfrac{\sqrt{7}}{3}\cdot\left(-\dfrac{2}{5}\right) + \dfrac{\sqrt{2}}{3}\cdot\dfrac{\sqrt{21}}{5} = \dfrac{2\sqrt{7}}{15} + \dfrac{\sqrt{42}}{15} = \dfrac{2\sqrt{7}+\sqrt{42}}{15}.$

331. $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \dfrac{\frac{1}{4}-3}{1-\frac{1}{4}\cdot(-3)} = \dfrac{1-12}{4+3} = -\dfrac{11}{7}.$

332. $\cos^2\alpha + \sin^2\alpha = 1$ より $\sin^2\alpha = 1 - \cos^2\alpha = 1 - \left(\dfrac{4}{5}\right)^2 = 1 - \dfrac{16}{25} = \dfrac{9}{25}$. $0 < \alpha < \dfrac{\pi}{2}$ より $\sin\alpha > 0$ だから

$\sin\alpha = \dfrac{3}{5}$. $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\frac{3}{5}}{\frac{4}{5}} = \dfrac{3}{4}$.

(1) 与式 $= 2\sin\alpha\cos\alpha = 2\cdot\dfrac{3}{5}\cdot\dfrac{4}{5} = \dfrac{24}{25}.$　　(2) 与式 $= \cos^2\alpha - \sin^2\alpha = \left(\dfrac{4}{5}\right)^2 - \left(\dfrac{3}{5}\right)^2 = \dfrac{7}{25}.$

(3) 与式 $\dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2\cdot\frac{3}{4}}{1-(\frac{3}{4})^2} = \dfrac{\frac{3}{2}}{1-\frac{9}{16}} = \dfrac{\frac{3}{2}}{\frac{7}{16}} = \dfrac{24}{7}.$

$\sin^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{2} = \dfrac{1-\frac{4}{5}}{2} = \dfrac{1}{10}, \cos^2\dfrac{\alpha}{2} = \dfrac{1+\cos\alpha}{2} = \dfrac{1+\frac{4}{5}}{2} = \dfrac{9}{10}, \tan^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{1+\cos\alpha} = \dfrac{1-\frac{4}{5}}{1+\frac{4}{5}} = \dfrac{1}{9}.$

$0 < \alpha < \dfrac{\pi}{2}$ より $2 < \dfrac{\alpha}{2} < \dfrac{\pi}{4}$ だから $\sin\dfrac{\alpha}{2}, \cos\dfrac{\alpha}{2}, \tan\dfrac{\alpha}{2} > 0$. よって

(4) $\sin\dfrac{\alpha}{2} = \dfrac{1}{\sqrt{10}} = \dfrac{\sqrt{10}}{10}.$　　(5) $\cos\dfrac{\alpha}{2} = \dfrac{3\sqrt{10}}{10}.$　　(6) $\tan\dfrac{\alpha}{2} = \dfrac{1}{3}.$

333. $\cos^2\alpha + \sin^2\alpha = \cos^2\beta + \sin^2\beta = 1$ より $\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\dfrac{2}{3}\right)^2 = 1 - \dfrac{4}{9} = \dfrac{5}{9}$,

$\sin^2\beta = 1 - \cos^2\beta = 1 - \left(-\dfrac{4}{5}\right)^2 = 1 - \dfrac{16}{25} = \dfrac{9}{25}$. α が第 2 象限の角, β が第 3 象限の角より

$\cos\alpha < 0, \sin\beta < 0$ だから $\cos\alpha = -\dfrac{\sqrt{5}}{3}, \sin\beta = -\dfrac{3}{5}$.

(1) 与式 $= \sin 2\alpha\cos\beta + \cos 2\alpha\sin\beta = 2\sin\alpha\cos\alpha\cos\beta + (\cos^2\alpha - \sin^2\alpha)\sin\beta$

$= 2\cdot\dfrac{2}{3}\cdot\left(-\dfrac{\sqrt{5}}{3}\right)\cdot\left(-\dfrac{4}{5}\right) + \left\{\left(-\dfrac{\sqrt{5}}{3}\right)^2 - \left(\dfrac{2}{3}\right)^2\right\}\cdot\left(-\dfrac{3}{5}\right) = \dfrac{16\sqrt{5}}{45} - \dfrac{5-4}{9}\cdot\dfrac{3}{5} = \dfrac{16\sqrt{5}-3}{45}.$

(2) 与式 $= \cos 2\alpha\cos\beta - \sin 2\alpha\sin\beta = (\cos^2\alpha - \sin^2\alpha)\cos\beta - 2\sin\alpha\cos\alpha\sin\beta$

$= \left\{\left(-\dfrac{\sqrt{5}}{3}\right)^2 - \left(\dfrac{2}{3}\right)^2\right\}\cdot\left(-\dfrac{4}{5}\right) - 2\cdot\dfrac{2}{3}\cdot\left(-\dfrac{\sqrt{5}}{3}\right)\cdot\left(-\dfrac{3}{5}\right) = -\dfrac{5-4}{9}\cdot\dfrac{4}{5} - \dfrac{4\sqrt{5}}{15} = -\dfrac{4+12\sqrt{5}}{45}.$

334. $\cos 2x = 1 - 2\sin^2 x$ より $1 - \cos 2x = 2\sin^2 x \cdots$①. $\cos 2x = 2\cos^2 x - 1$ より $1 + \cos 2x = 2\cos^2 x \cdots$②. ①②より

左辺 $= \dfrac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x} = \dfrac{2\sin^2 x+\sin 2x}{2\cos^2 x+\sin 2x} = \dfrac{2\sin^2 x+2\sin x\cos x}{2\cos^2 x+2\sin x\cos x} = \dfrac{2\sin x(\sin x+\cos x)}{2\cos x(\cos x+\sin x)} = \dfrac{\sin x}{\cos x}$

$= \tan x =$ 右辺//

335. (1) 与式 $= 2\cdot\dfrac{1}{2}\{\sin((\theta+120^\circ)+(30^\circ-\theta))+\sin((\theta+120^\circ)-(30^\circ-\theta))\} = \sin 150^\circ + \sin(2\theta+90^\circ)$

$= \sin 30^\circ + \sin 2\theta\cos 90^\circ + \cos 2\theta \sin 90^\circ = \cos 2\theta + \dfrac{1}{2}$.

(2) 与式 $= \dfrac{1}{2}\left\{\cos\left(\dfrac{2\theta+3\pi}{4}+\dfrac{2\theta-3\pi}{4}\right)+\cos\left(\dfrac{2\theta+3\pi}{4}-\dfrac{2\theta-3\pi}{4}\right)\right\} = \dfrac{1}{2}\left(\cos\theta+\cos\dfrac{3}{2}\pi\right) = \dfrac{1}{2}\cos\theta$.

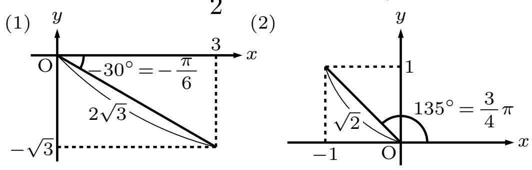
336. (1) 与式 $= 2\sin\dfrac{100^\circ+40^\circ}{2}\cos\dfrac{100^\circ-40^\circ}{2} = 2\sin 70^\circ\cos 30^\circ = 2\sin 70^\circ\cdot\dfrac{\sqrt{3}}{2} = \sqrt{3}\sin 70^\circ$.

(2) 与式 $= -2\sin\dfrac{100^\circ+20^\circ}{2}\sin\dfrac{100^\circ-20^\circ}{2} = -2\sin 60^\circ\sin 40^\circ = -2\cdot\dfrac{\sqrt{3}}{2}\sin 40^\circ = -\sqrt{3}\sin 40^\circ$.

337. (1) $\sqrt{3^2+(-\sqrt{3})^2} = 2\sqrt{3}$ より $y = 2\sqrt{3}\sin\left(x-\dfrac{\pi}{6}\right)$.

$y=\sin x$ のグラフを y 軸方向に $2\sqrt{3}$ 倍に拡大し,

x 軸方向に $\dfrac{\pi}{6}$ 平行移動 (グラフは下図).



(2) $\sqrt{(-1)^2+1^2} = \sqrt{2}$ より $y = \sqrt{2}\sin\left(x+\dfrac{3}{4}\pi\right)$. $y=\sin x$ のグラフを y 軸方向に $\sqrt{2}$ 倍に拡大し, x 軸方向に $-\dfrac{3}{4}\pi$ 平行移動 (グラフは下図).