大日本図書　新基礎数学

1 章§ 2. いろいろな数と式

p.32.　練習問題 2-A

1. (1) 与式 $= \dfrac{9x^2y^6}{-8x^6y^3} = -\dfrac{9y^3}{8x^4}$.

(2) 与式 $= \dfrac{x(x-y)}{(x+y)(x-y)} + \dfrac{y(x+y)}{(x-y)(x+y)} - \dfrac{x^2+y^2}{(x+y)(x-y)} = \dfrac{(x^2-xy)+(xy+y^2)-(x^2+y^2)}{(x+y)(x-y)} = 0$.

(3) 与式 $= \dfrac{(x+y)-y}{x(x+y)} - \dfrac{z}{(x+y)(x+y+z)} = \dfrac{x}{x(x+y)} - \dfrac{z}{(x+y)(x+y+z)} = \dfrac{1}{x+y} - \dfrac{z}{(x+y)(x+y+z)}$
$= \dfrac{(x+y+z)-z}{(x+y)(x+y+z)} = \dfrac{x+y}{(x+y)(x+y+z)} = \dfrac{1}{x+y+z}$.

(4) 与式 $= \dfrac{(a+2)(a-3)}{(a+4)(a-3)} \times \dfrac{(a+4)(a-4)}{(a+2)(a-2)} \times \dfrac{a-2}{a-4} = 1$.

(5) 与式 $= \dfrac{\left\{\dfrac{a^2+1}{(a+1)(a-1)} - 1\right\} \times (a+1)(a-1)}{\left(\dfrac{a-1}{a+1} - \dfrac{a+1}{a-1}\right) \times (a+1)(a-1)} = \dfrac{a^2+1-(a+1)(a-1)}{(a-1)^2-(a+1)^2} = \dfrac{a^2+1-(a^2-1)}{(a^2-2a+1)-(a^2+2a+1)}$
$= \dfrac{2}{-4a} = -\dfrac{1}{2a}$.

2. (1) 与式 $= \dfrac{\sqrt{3}+1}{\sqrt{3}-1} + \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{(\sqrt{3}+1)^2+(\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \dfrac{3+2\sqrt{3}+1+3-2\sqrt{3}+1}{3-1} = \dfrac{8}{2} = 4$.

(2) 与式 $= \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 1$.

(3) (1), (2) より与式 $= (x+y)^2 - 2xy = 4^2 - 2 \times 1 = 16 - 2 = 14$.

(4) (1), (2),(3) より与式 $= (x+y)(x^2-xy+y^2) = 4 \times (14-1) = 52$.

3. (1) 与式 $= \{\sqrt{5} + (\sqrt{3}-\sqrt{2})\}\{\sqrt{5} - (\sqrt{3}-\sqrt{2})\} = \sqrt{5}^2 - (\sqrt{3}-\sqrt{2})^2 = 5 - (3 - 2\sqrt{6} + 2) = 2\sqrt{6}$.

(2) 与式 $= \dfrac{(1-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} - \dfrac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \dfrac{2-\sqrt{3}-2\sqrt{3}+3}{4-3} - \dfrac{3+2\sqrt{3}+1}{3-1} = 5 - 3\sqrt{3} - \dfrac{4+2\sqrt{3}}{2}$
$= 5 - 3\sqrt{3} - 2 - \sqrt{3} = 3 - 4\sqrt{3}$.

(3) 与式 $= \dfrac{2+\sqrt{3}i}{2-\sqrt{3}i} + \dfrac{2-\sqrt{3}i}{2+\sqrt{3}i} = \dfrac{(2+\sqrt{3}i)^2}{(2-\sqrt{3}i)(2+\sqrt{3}i)} + \dfrac{(2-\sqrt{3}i)^2}{(2+\sqrt{3}i)(2-\sqrt{3}i)} = \dfrac{4+4\sqrt{3}i+3i^2+4-4\sqrt{3}i+3i^2}{4-3i^2}$
$= \dfrac{4+4\sqrt{3}i-3+4-4\sqrt{3}i-3}{4+3} = \dfrac{2}{7}$.

(4) 与式 $= \dfrac{(\sqrt{2}+i)^2}{(\sqrt{2}-i)(\sqrt{2}+i)} - \dfrac{(\sqrt{2}-i)^2}{(\sqrt{2}+i)(\sqrt{2}-i)} = \dfrac{(2+2\sqrt{2}i+i^2)-(2-2\sqrt{2}i+i^2)}{2-i^2}$
$= \dfrac{(2+2\sqrt{2}i-1)-(2-2\sqrt{2}i-1)}{2+1} = \dfrac{4\sqrt{2}i}{3}$.

4. (1) 与式 $= \dfrac{|\sqrt{5}-3|}{|-2+\sqrt{5}|} = \dfrac{-(\sqrt{5}-3)}{-2+\sqrt{5}} = \dfrac{(3-\sqrt{5})(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)} = \dfrac{3\sqrt{5}+6-5-2\sqrt{5}}{5-4} = 1+\sqrt{5}$ $(2 < \sqrt{5} < 3$ に注意$)$

(2) 与式 $= \sqrt{(-2)^2+\sqrt{2}^2} - \sqrt{(-\sqrt{5})^2+(-1)^2} = \sqrt{6} - \sqrt{6} = 0$.

(3) 与式 $= \dfrac{|\sqrt{2}+3i|}{|2-\sqrt{3}i|} = \dfrac{\sqrt{\sqrt{2}^2+3^2}}{\sqrt{2^2+(-\sqrt{3})^2}} = \dfrac{\sqrt{11}}{\sqrt{7}} = \dfrac{\sqrt{77}}{7}$.

p. 33 練習問題 2-B

1. (1) 与式 $= \dfrac{2a^2}{(2a+b)(2a-b)} + \dfrac{a-b}{-2a+b} = \dfrac{2a^2}{(2a+b)(2a-b)} - \dfrac{(a-b)(2a+b)}{(2a-b)(2a+b)} = \dfrac{2a^2-(2a^2-ab-b^2)}{(2a+b)(2a-b)}$
$= \dfrac{ab+b^2}{(2a+b)(2a-b)} = \dfrac{b(a+b)}{(2a+b)(2a-b)}$.

(2) 与式 $= \dfrac{(a+1)-(a-1)}{(a-1)(a+1)} - \dfrac{2}{a^2+1} - \dfrac{4}{a^4+1} = \dfrac{2}{a^2-1} - \dfrac{2}{a^2+1} - \dfrac{4}{a^4+1} = \dfrac{2(a^2+1)-2(a^2-1)}{(a^2-1)(a^2+1)} - \dfrac{4}{a^4+1}$

$= \dfrac{4}{a^4-1} - \dfrac{4}{a^4+1} = \dfrac{4(a^4+1)-4(a^4-1)}{(a^4-1)(a^4+1)} = \dfrac{8}{a^8-1}.$

(3) 与式 $= \dfrac{x^3}{x + \dfrac{1\times x}{\left(x-\dfrac{1}{x}\right)\times x}} = \dfrac{x^3}{x+\dfrac{x}{x^2-1}} = \dfrac{x^3\times(x^2-1)}{\left(x+\dfrac{x}{x^2-1}\right)\times(x^2-1)} = \dfrac{x^3(x^2-1)}{x(x^2-1)+x} = \dfrac{x^3(x^2-1)}{x^3-x+x}$

$= \dfrac{x^3(x^2-1)}{x^3} = x^2-1.$

(4) 与式 $= \dfrac{(x+2)\times(x+3)}{\left(1-\dfrac{1}{x+3}\right)\times(x+3)} - \dfrac{(x+2)\times(x+1)}{\left(1+\dfrac{1}{x+1}\right)\times(x+1)} = \dfrac{(x+2)(x+3)}{x+3-1} - \dfrac{(x+2)(x+1)}{x+1+1}$

$= \dfrac{(x+2)(x+3)}{x+2} - \dfrac{(x+2)(x+1)}{x+2} = x+3-(x+1) = 2.$

(5) 与式 $= \dfrac{2a}{\dfrac{1\times a}{\left(1-\dfrac{1}{a}\right)\times a} - \dfrac{1\times a}{\left(1+\dfrac{1}{a}\right)\times a}} = \dfrac{2a}{\dfrac{a}{a-1}-\dfrac{a}{a+1}} = \dfrac{2a\times(a-1)(a+1)}{\left(\dfrac{a}{a-1}-\dfrac{a}{a+1}\right)\times(a-1)(a+1)}$

$= \dfrac{2a(a-1)(a+1)}{a(a+1)-a(a-1)} = \dfrac{2a(a-1)(a+1)}{a^2+a-a^2+a} = \dfrac{2a(a-1)(a+1)}{2a} = (a-1)(a+1) = a^2-1.$

2. (1) 与式 $= \dfrac{\sqrt{2}-\sqrt{1}}{(\sqrt{2}+\sqrt{1})(\sqrt{2}-\sqrt{1})} + \dfrac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} + \dfrac{\sqrt{4}-\sqrt{3}}{(\sqrt{4}+\sqrt{3})(\sqrt{4}-\sqrt{3})}$

$= \dfrac{\sqrt{2}-\sqrt{1}}{2-1} + \dfrac{\sqrt{3}-\sqrt{2}}{3-2} + \dfrac{\sqrt{4}-\sqrt{3}}{4-3} = \sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3} = \sqrt{4}-\sqrt{1} = 2-1 = 1.$

(2) 与式 $= \dfrac{(1+\sqrt{2})-\sqrt{3}}{\{(1+\sqrt{2})+\sqrt{3}\}\{(1+\sqrt{2})-\sqrt{3}\}} = \dfrac{1+\sqrt{2}-\sqrt{3}}{(1+\sqrt{2})^2-\sqrt{3}^2} = \dfrac{1+\sqrt{2}-\sqrt{3}}{1+2\sqrt{2}+2-3} = \dfrac{1+\sqrt{2}-\sqrt{3}}{2\sqrt{2}}$

$= \dfrac{\sqrt{2}+2-\sqrt{6}}{4}.$

3. $\sqrt{a^2-x^2} = \sqrt{a^2-(2\sqrt{a-1})^2} = \sqrt{a^2-\{4(a-1)\}} = \sqrt{a^2-4a+4} = \sqrt{(a-2)^2} = |a-2|.$

よって $a \geqq 2$ のとき $a-2 \geqq 0$ だから $|a-2| = a-2$ より $\sqrt{a^2-x^2} = a-2.$

$1 \leqq a < 2$ のとき $a-2<0$ だから $|a-2| = -(a-2) = 2-a$ より $\sqrt{a^2-x^2} = 2-a.$

4. $\alpha = a+bi, \beta = c+di$ (a,b,c,d は実数) とおいて証明する.

(1) $\overline{\alpha+\beta} = \overline{(a+bi)+(c+di)} = \overline{(a+c)+(b+d)i} = (a+c)-(b+d)i.$

$\overline{\alpha}+\overline{\beta} = (a-bi)+(c-di) = (a+c)-(b+d)i.$ よって $\overline{\alpha+\beta} = \overline{\alpha}+\overline{\beta}.$

(2) $\overline{\alpha-\beta} = \overline{(a+bi)-(c+di)} = \overline{(a-c)+(b-d)i} = (a-c)-(b-d)i.$

$\overline{\alpha}-\overline{\beta} = (a-bi)-(c-di) = (a-c)+(-b+d)i = (a-c)-(b-d)i.$ よって $\overline{\alpha-\beta} = \overline{\alpha}-\overline{\beta}.$

(3) $\overline{\alpha\beta} = \overline{(a+bi)(c+di)} = \overline{ac+adi+bci+bdi^2} = \overline{(ac-bd)+(ad+bc)i} = (ac-bd)-(ad+bc)i.$

$\overline{\alpha}\overline{\beta} = (a-bi)(c-di) = ac-adi-bci+bdi^2 = (ac-bd)-(ad+bc)i.$ よって $\overline{\alpha\beta} = \overline{\alpha}\overline{\beta}.$

(4) $\overline{\left(\dfrac{\alpha}{\beta}\right)} = \overline{\left(\dfrac{a+bi}{c+di}\right)} = \overline{\left\{\dfrac{(a+bi)(c-di)}{(c+di)(c-di)}\right\}} = \overline{\left(\dfrac{ac-adi+bci-bdi^2}{c^2-d^2i^2}\right)} = \overline{\left\{\dfrac{(ac+bd)-(ad-bc)i}{c^2+d^2}\right\}}$

$= \overline{\left(\dfrac{ac+bd}{c^2+d^2} - \dfrac{ad-bc}{c^2+d^2}i\right)} = \dfrac{ac+bd}{c^2+d^2} + \dfrac{ad-bc}{c^2+d^2}i.$

$\dfrac{\overline{\alpha}}{\overline{\beta}} = \dfrac{a-bi}{c-di} = \dfrac{(a-bi)(c+di)}{(c-di)(c+di)} = \dfrac{ac+adi-bci-bdi^2}{c^2-d^2i^2} = \dfrac{(ac+bd)+(ad-bc)i}{c^2+d^2} = \dfrac{ac+bd}{c^2+d^2} + \dfrac{ad-bc}{c^2+d^2}i.$

よって $\overline{\left(\dfrac{\alpha}{\beta}\right)} = \dfrac{\overline{\alpha}}{\overline{\beta}}.$