Converted from "**Complex Numbers in Mathcad**" created by **Mathcad Staff**

Working with Complex Numbers

A complex number is an expression of the form

$$x+y\cdot 1\mathrm{i}$$

where x and y are real numbers, and

$$1\mathrm{i}=\sqrt{-1}$$

For example, the following is a complex number:

$$2+13\mathrm{i} \qquad \overline{2+13\mathrm{i}}=2-13\mathrm{i} \qquad \overline{2+13\mathrm{i}}\cdot(2+13\mathrm{i})=173$$

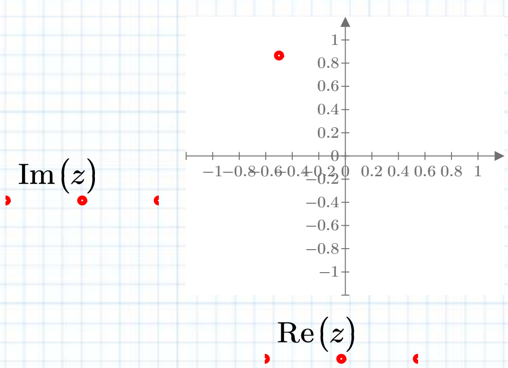
Real and Imaginary Parts

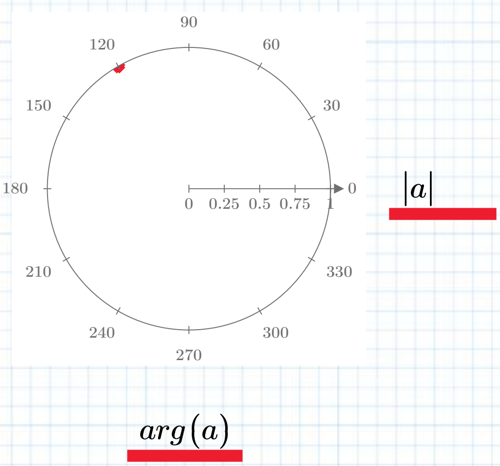
$$a:=-\frac{1}{2}+\frac{\sqrt{3}}{2}\,1\mathrm{i} \qquad a^2=-0.5-0.866\mathrm{i} \qquad a^3=1+1.11\mathrm{i}\cdot 10^{-16} \qquad a^3 \xrightarrow{simplify} 1$$

$$\mathrm{Re}(a)=-0.5 \qquad \mathrm{Im}(a)=0.866 \qquad \arg(a)=2.094 \qquad \arg(a) \xrightarrow{simplify} \frac{2\cdot\pi}{3} \qquad |a|=1$$

Representing Complex Numbers as Points in the Plane

$$z:=-\frac{1}{2}+\frac{\sqrt{3}}{2}\,1\mathrm{i}$$





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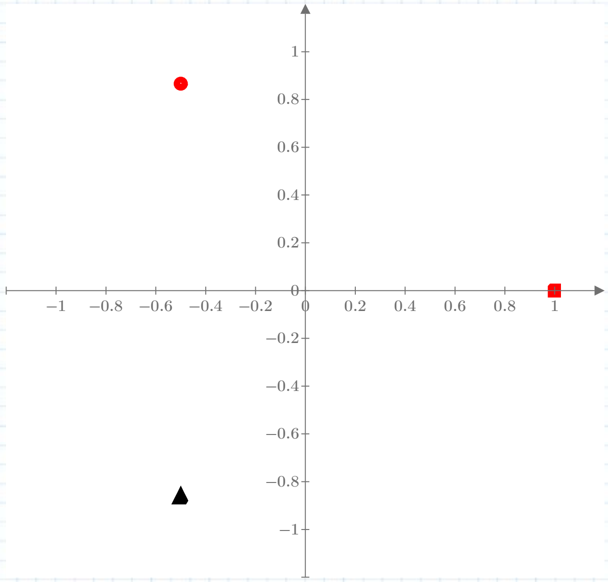
$$z := -\frac{1}{2} + \frac{\sqrt{3}}{2}\,1i$$



$$\mathrm{Im}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\,1i\right)$$

$$\mathrm{Im}\left(\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\,1i\right)^2\right)$$

$$\mathrm{Im}\left(\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\,1i\right)^3\right)$$

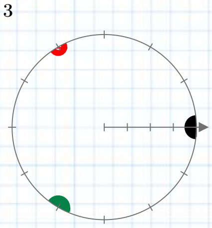
$$\mathrm{Re}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\,1i\right)$$

$$\mathrm{Re}\left(\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\,1i\right)^2\right)$$

$$\mathrm{Re}\left(\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\,1i\right)^3\right)$$

$$e^{\frac{\pi}{3}\cdot 1i} \rightarrow \frac{1}{2} + \frac{\sqrt{3}\cdot 1i}{2}$$

$$e^{\frac{2\cdot\pi}{3}\cdot 1i}$$



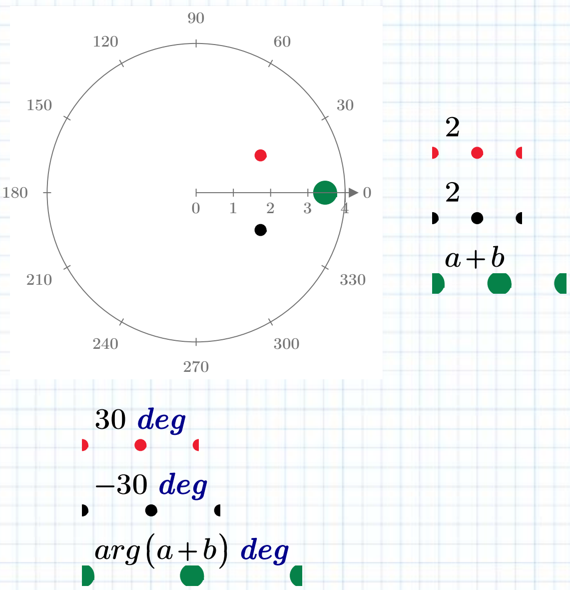
$$e^{\frac{0\cdot\pi}{3}\cdot 1i}$$

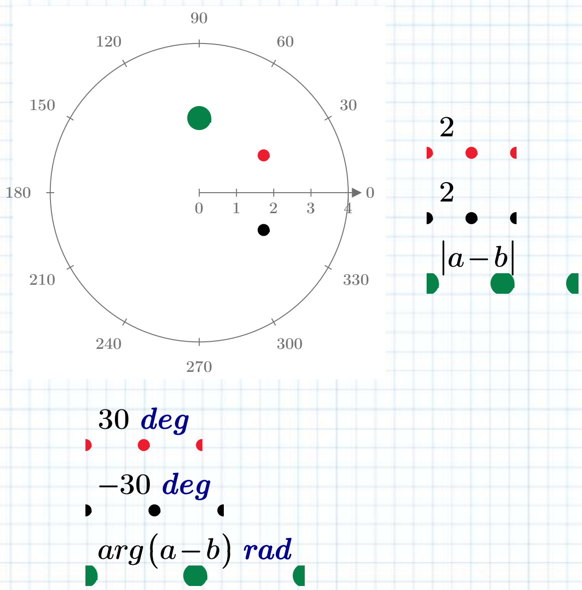
$$e^{\frac{4\cdot\pi}{3}\cdot 1i}$$

[redacted]

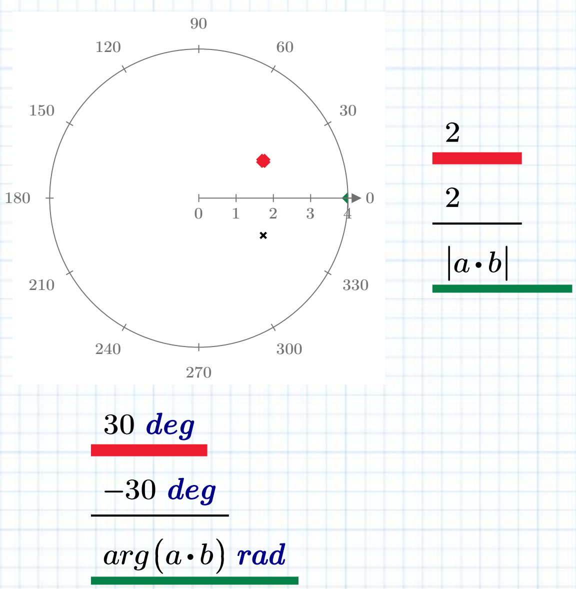
$a := 2\angle\frac{\pi}{6}$ $\qquad$ $b := 2\angle\frac{-\pi}{6}$

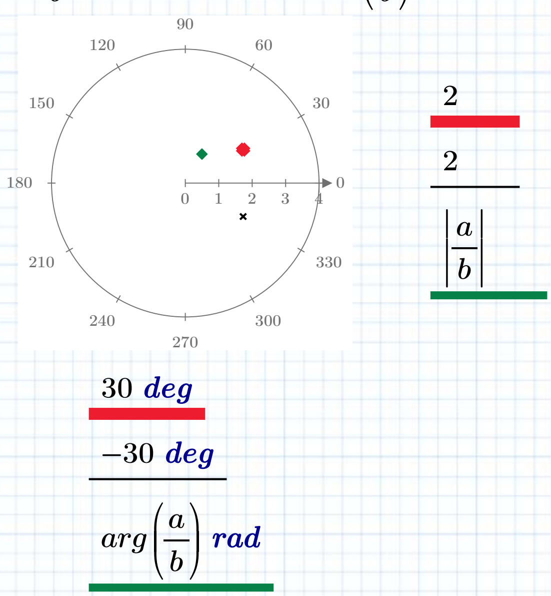
$a+b \xrightarrow{simplify} 2\cdot\sqrt{3}$ $\qquad$ $\arg(a+b)=0$ $\qquad$ $a-b=2i$ $\qquad$ $\arg(a-b)=1.571$





$a\cdot b=4$ $\qquad$ $\arg(a\cdot b)=0$ $\qquad$ $\frac{a}{b}=0.5+0.866i$ $\qquad$ $\arg\left(\frac{a}{b}\right)=1.047$





If you square a complex number z on the unit circle, the magnitude of is 1, since

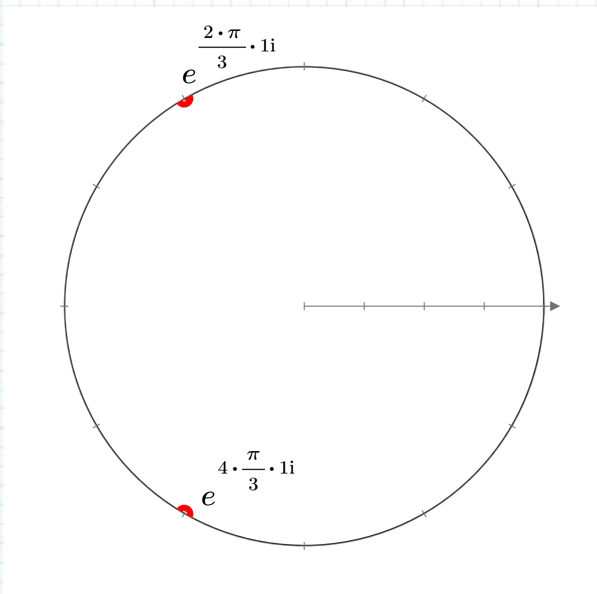
$$|z^2| = |z| \cdot |z| = 1 \cdot 1 = 1$$

The argument of z^2 is twice the argument of z, since

$$arg(z^2) = arg(z) \cdot arg(z) = 2 \cdot arg(z)$$

For example,

$$\left(e^{\frac{2\cdot\pi}{3}\cdot 1i}\right)^2 \xrightarrow{expand} -\frac{1}{2} - \frac{\sqrt{3}\cdot 1i}{2}$$



Roots of Unity

In the complex numbers, the equation

$$z^n = 1$$

has n distinct solutions, called the *nth roots of unity*. These are defined as follows. Let

$$\alpha = e^{\frac{2\cdot\pi}{n}\cdot 1i} = \cos\left(\frac{2\cdot\pi}{n}\right) + \sin\left(\frac{2\cdot\pi}{n}\right)\cdot 1i$$

α corresponds to the point on the unit circle in the complex plane whose angle is 2п/n.

Note that α is a solution to the equation

$$x^n = 1$$

since

$$\alpha^n = \left(e^{\frac{2\cdot\pi}{n}\cdot 1i}\right)^n = e^{2\cdot\pi\cdot 1i} = \cos(2\cdot\pi) + \sin(2\cdot\pi)\cdot 1i = 1$$

The nth roots of unity are the numbers

$$\alpha, \alpha^2 .. \alpha^n = 1$$

For example, if n = 3, the cube roots of unity are the numbers

$$\alpha := e^{\frac{2\cdot\pi}{3}\cdot 1i} \rightarrow -\frac{1}{2} + \frac{\sqrt{3}\cdot 1i}{2}$$

$$\alpha^2 \xrightarrow{expand} -\frac{1}{2} - \frac{\sqrt{3}\cdot 1i}{2}$$

$$\alpha^3 \xrightarrow{expand} 1$$

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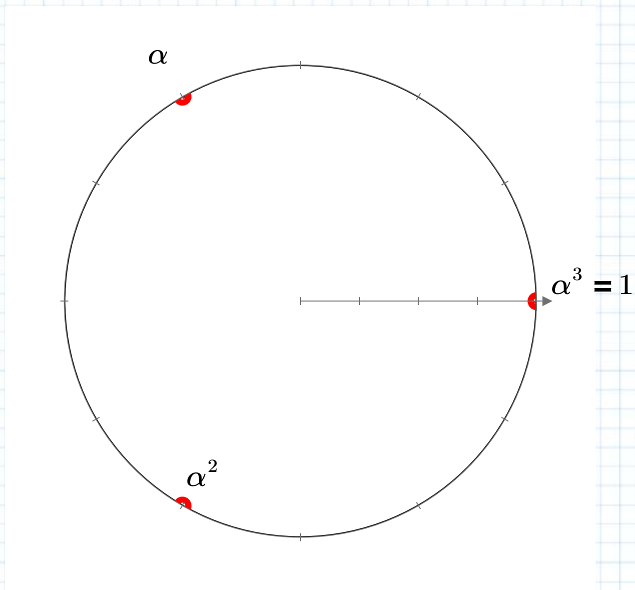
Since raising α to any power k multiplies the argument of α by k, the powers of α are evenly spaced around the unit circle. The following graph shows the case n = 3.



Note that each of these numbers is a solution to the equation

$$x^3 = 1$$

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