問題１． $L\{\sin \omega t\} = \int_0^\infty \sin \omega t e^{-st} dt \quad (s>0)$, $L\{\cos \omega t\} = \int_0^\infty \cos \omega t e^{-st} dt \quad (s>0)$ を求めよ。

$$\sin(\omega \cdot t) \xrightarrow{laplace} \frac{\omega}{\omega^2 + s^2} \qquad \cos(\omega \cdot t) \xrightarrow{laplace} \frac{s}{\omega^2 + s^2}$$

$$e^{1i \cdot \omega \cdot t} \xrightarrow{rectangular} \cos(\omega \cdot t) + \sin(\omega \cdot t) \cdot 1i \qquad e^{-\alpha \cdot t} \xrightarrow{laplace} \frac{1}{\alpha + s}$$

$$e^{1i \cdot \omega \cdot t} \xrightarrow{laplace} \frac{1}{s - \omega \cdot 1i} \xrightarrow{rectangular} \frac{s}{\omega^2 + s^2} + \frac{\omega}{\omega^2 + s^2} \cdot 1i$$

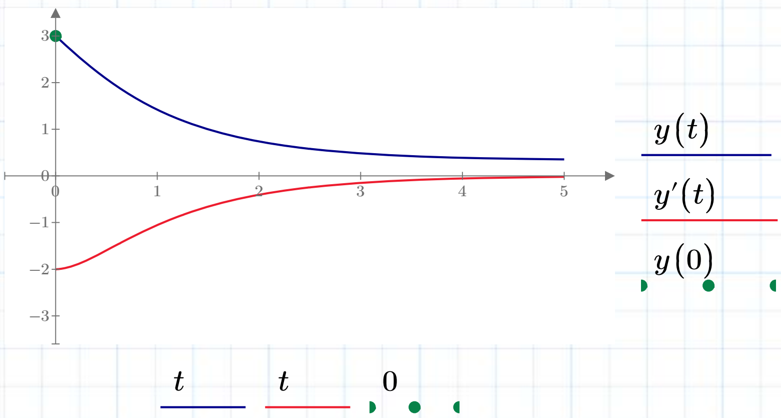
問題２． $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 1 \quad (t = 0 ; y = 3, \frac{dy}{dt} = -2)$ を求めよ。

$$y'' + 4 \cdot y' + 3 \cdot y = 1 \qquad y(0) = 3 \qquad y'(0) = -2 \qquad 1 \xrightarrow{laplace} \frac{1}{s}$$

$$s^2 \cdot Y(s) - s \cdot y(0) - y'(0) + 4 \cdot (s \cdot Y(s) - y(0)) + 3 \cdot Y(s) = \frac{1}{s} \xrightarrow[solve, Y(s)]{substitute, y(0)=3 \\ substitute, y'(0)=-2} \frac{3 \cdot s + \frac{1}{s} + 10}{s^2 + 4 \cdot s + 3}$$

$$\frac{3 \cdot s + \frac{1}{s} + 10}{s^2 + 4 \cdot s + 3} \xrightarrow{invlaplace} 3 \cdot e^{-t} - \frac{e^{-3 \cdot t}}{3} + \frac{1}{3} \qquad y(t) := 3 \cdot e^{-t} - \frac{e^{-3 \cdot t}}{3} + \frac{1}{3}$$

$$y(t) := s^2 \cdot Y(s) - s \cdot y(0) - y'(0) + 4 \cdot (s \cdot Y(s) - y(0)) + 3 \cdot Y(s) = \frac{1}{s} \xrightarrow{substitute, y(0)=3 \\ substitute, y'(0)=-2 \\ solve, Y(s) \\ invlaplace} 3 \cdot e^{-t} - \frac{e^{-3 \cdot t}}{3} +$$



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$t := 0, 0.01 .. 6$

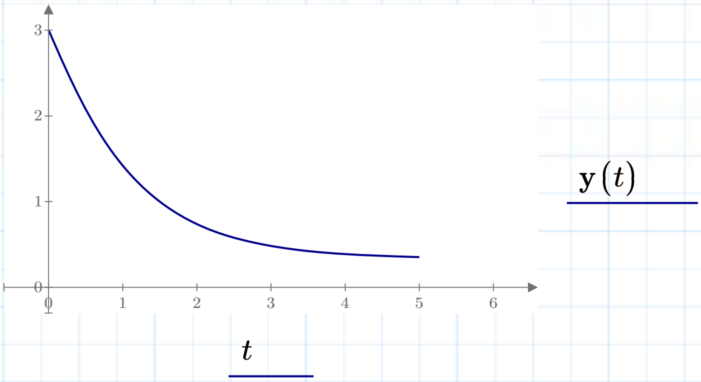
推定値

制約条件

$$\frac{d^2}{dt^2} y(t) + 4 \cdot \frac{d}{dt} y(t) + 3 \cdot y(t) = 1 \qquad y'(0) = -2 \qquad y(0) = 3$$

ソルバー

$$\mathbf{y} := \mathbf{odesolve}(y(t), 5)$$



例題３．次の連立微分方程式を解け

$$\begin{cases} \frac{dx}{dt} - 7x + y = 0 \\ \frac{dy}{dt} - 2x - 5y = 0 \end{cases}$$

$x(0) = 1,\ y(0) = -1$

$$\begin{cases} (s-7)X(s) + Y(s) = 1 \\ -2X(s) + (s-5)Y(s) = -1 \end{cases}$$

$$\begin{cases} X(s) = \frac{s-4}{s^2 - 12s + 37} \\ Y(s) = \frac{-s+9}{s^2 - 12s + 37} \end{cases}$$

$t := 0, 0.01 .. 1$

推定値

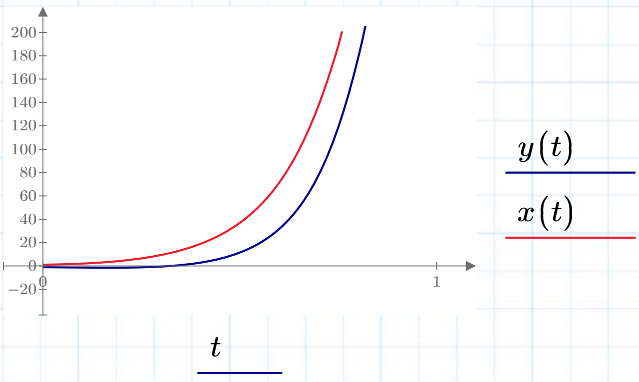
制約条件

$$\frac{d}{dt} y(t) - 2 \cdot x(t) - 5 \cdot y(t) = 0$$

$$\frac{d}{dt} x(t) - 7 \cdot x(t) + y(t) = 0 \qquad x(0) = 1 \qquad y(0) = -1$$

ソルバー

$$\begin{bmatrix} x \\ y \end{bmatrix} := \mathbf{odesolve}\left(\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, 1 \right)$$



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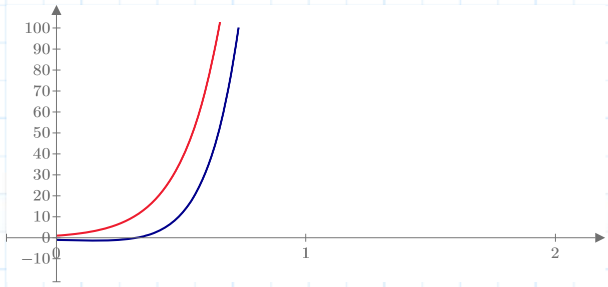
$$s \cdot X(s) - 1 - 7 \cdot X(s) + Y(s) = 0 \xrightarrow{solve, Y(s)} 7 \cdot X(s) - s \cdot X(s) + 1$$

$$s \cdot Y(s) + 1 - 2 \cdot X(s) - 5 \cdot Y(s) = 0 \xrightarrow{substitute, Y(s) = 7 \cdot X(s) - s \cdot X(s) + 1} s - 37 \cdot X(s) + 12 \cdot s \cdot X(s) - s^2 \cdot$$

$$s - 37 \cdot X(s) + 12 \cdot s \cdot X(s) - s^2 \cdot X(s) - 4 = 0 \xrightarrow{solve, X(s)} \frac{s-4}{s^2 - 12 \cdot s + 37} \xrightarrow{invlaplace} e^{6 \cdot t} \cdot (\cos(t) + 2 \cdot \sin($$

$$7 \cdot X(s) - s \cdot X(s) + 1 \xrightarrow{substitute, X(s) = \frac{s-4}{s^2 - 12 \cdot s + 37}} -\frac{s-9}{s^2 - 12 \cdot s + 37} \xrightarrow{invlaplace} -\left(e^{6 \cdot t} \cdot (\cos(t) - 3 \cdot s\right.$$

$t := 0, 0.01 .. 2$



$-\left(e^{6 \cdot t} \cdot (\cos(t) - 3 \cdot \sin(t))\right)$

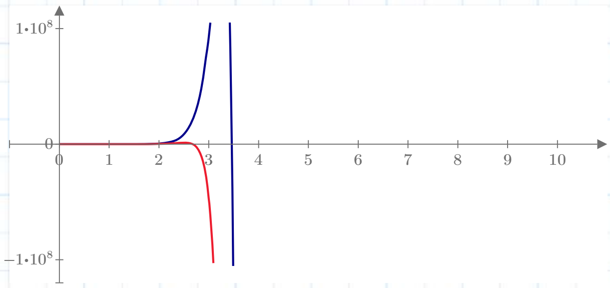
$e^{6 \cdot t} \cdot (\cos(t) + 2 \cdot \sin(t))$

t

$\text{clear}(s, x, y, t)$

$$f(t) := \begin{bmatrix} (s \cdot X(s) - x(0)) - 7 \cdot X(s) + Y(s) = 0 \\ (s \cdot Y(s) - y(0)) - 2 \cdot X(s) - 5 \cdot Y(s) = 0 \end{bmatrix} \xrightarrow[]{\substack{substitute, x(0) = 1 \\ substitute, y(0) = -1 \\ solve, [X(s) \;\; Y(s)] \\ invlaplace}} \left[e^{6 \cdot t} \cdot (\cos(t) + 2 \cdot \sin(t)) \;\; -\left(e^{6 \cdot t} \cdot\right.\right.$$

$x(t) := f(t)_0 \rightarrow e^{6 \cdot t} \cdot (\cos(t) + 2 \cdot \sin(t))$ $\quad y(t) := f(t)_1 \rightarrow -\left(e^{6 \cdot t} \cdot (\cos(t) - 3 \cdot \sin(t))\right)$



$-\left(e^{6 \cdot t} \cdot (\cos(t) - 3 \cdot \sin(t))\right)$

$e^{6 \cdot t} \cdot (\cos(t) + 2 \cdot \sin(t))$

t

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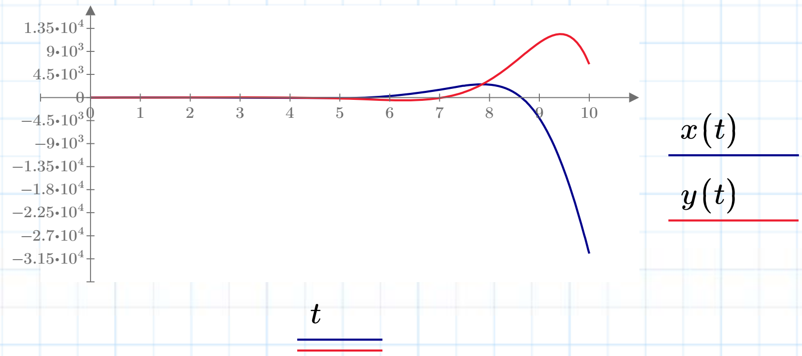
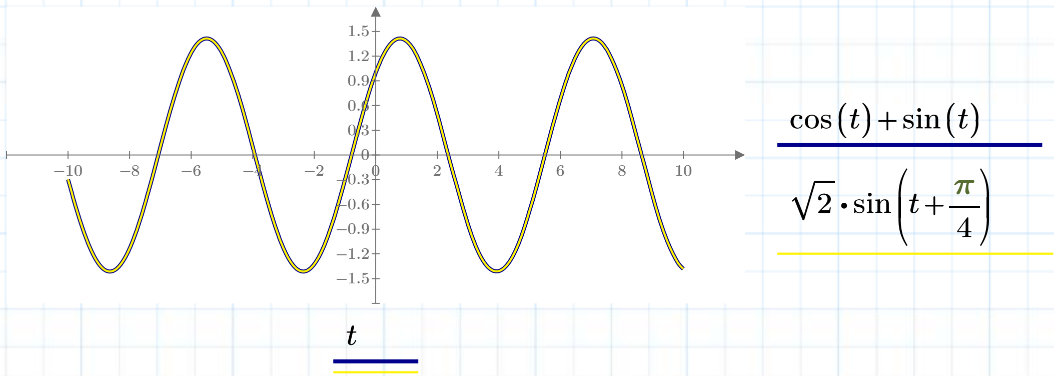
$$\mathbf{clear}(s, x, y, t)$$

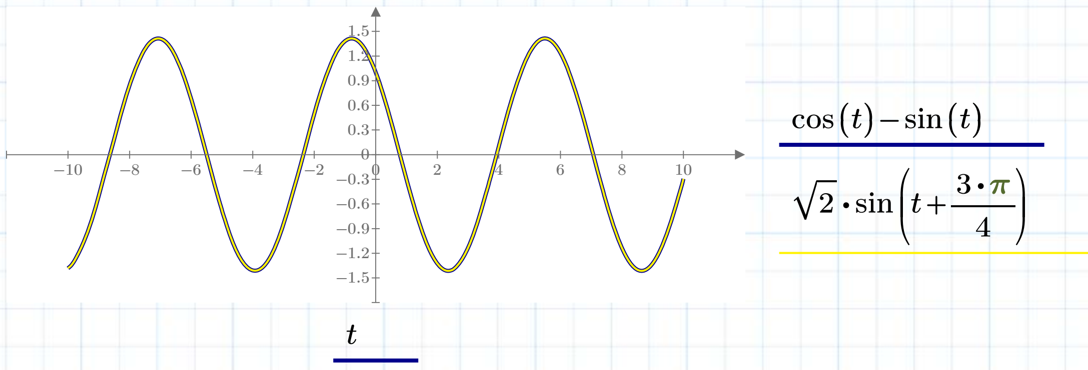
$$f(t) := \begin{bmatrix} (s \cdot X(s) - x(0)) - 1 \cdot X(s) + Y(s) = 0 \\ (s \cdot Y(s) - y(0)) - 1 \cdot X(s) - 1 \cdot Y(s) = 0 \end{bmatrix} \xrightarrow[\substack{\textit{substitute}, x(0) = 1 \\ \textit{substitute}, y(0) = -1 \\ \textit{solve}, [X(s)\ \ Y(s)] \\ \textit{invlaplace}}]{} \left[e^{t} \cdot (\cos(t) + \sin(t)) \quad -\left(e^{t} \cdot (\cos(\right.\right.$$

$$x(t) := f(t)_0 \rightarrow e^{t} \cdot (\cos(t) + \sin(t))$$

$$y(t) := f(t)_1 \rightarrow -\left(e^{t} \cdot (\cos(t) - \sin(t))\right)$$







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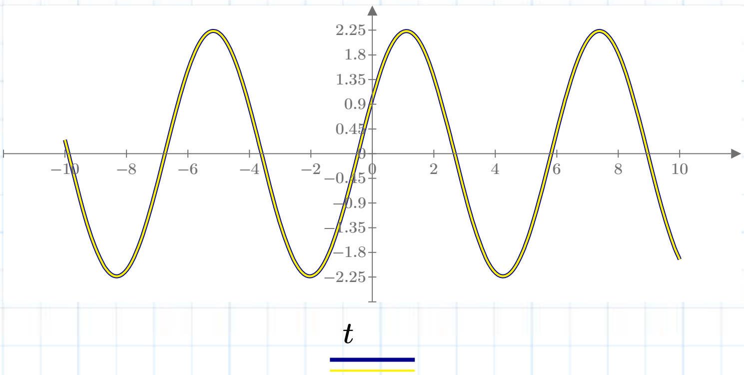
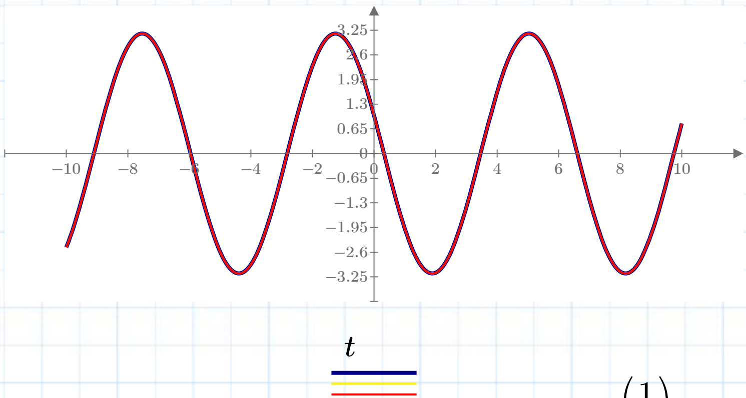


$\cos(t)+2\cdot\sin(t)$

$\sqrt{1^2+2^2}\cdot\sin\left(t+\operatorname{atan}\left(\frac{1}{2}\right)\right)$



$\cos(t)-3\cdot\sin(t)$

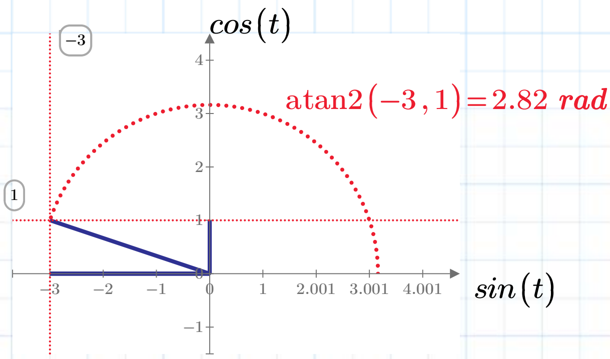
$\sqrt{1^2+3^2}\cdot\sin(t+\operatorname{atan2}(-3,1))$

$\sqrt{1^2+3^2}\cdot\cos\left(t+\operatorname{atan}\left(\frac{3}{1}\right)\right)$

$\pi-\operatorname{atan}\left(\frac{1}{3}\right)=2.82$ $\operatorname{atan2}(-3,1)=2.82$

$x_1:=-3$ $y_1:=0$ $x_3:=0$ $y_3:=1$ $x_5:=-3$ $y_5:=1$

$\theta:=0,0.01..2.82$ $C(\theta):=\sqrt{10}\cdot e^{1i\cdot\theta}$



sinとcosの合成

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