Lesson１４． 微分方程式をMathcadで解く

情報伝送工学や微分方程式の例題をMathcadで解いてみる。

① $y''-3y'+2y=\sin t \quad y(0)=0,\ y'(0)=1$

② $\dfrac{dy}{dx}-y=x^2 \quad x=1, y=1$

Q.14-1　上記の微分方程式を解きグラフ表示せよ。

推定値

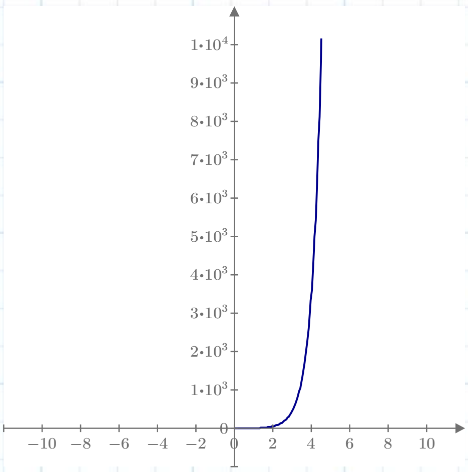
$y:=0$

制約条件

$y''(t)-3\cdot y'(t)+2\cdot y(t)=\sin(t) \qquad y(0)=0 \qquad y'(0)=1$

ソルバー

$y:=\mathbf{odesolve}(y(t),10)$



$y(t)$

t

① $y''-3y'+2y=\sin t \quad y(0)=0,\ y'(0)=1$

Q.14-2　上記の微分方程式をLaplace変換を用いて解きグラフ表示せよ。

$$\sin(t) \xrightarrow{laplace} \frac{1}{s^2+1}$$

$$\left(s^2 \cdot Y(s) - s \cdot y(0) - y'(0)\right) - 3 \cdot \left(s \cdot Y(s) - y(0)\right) + 2 \cdot Y(s) = \frac{1}{s^2+1}$$

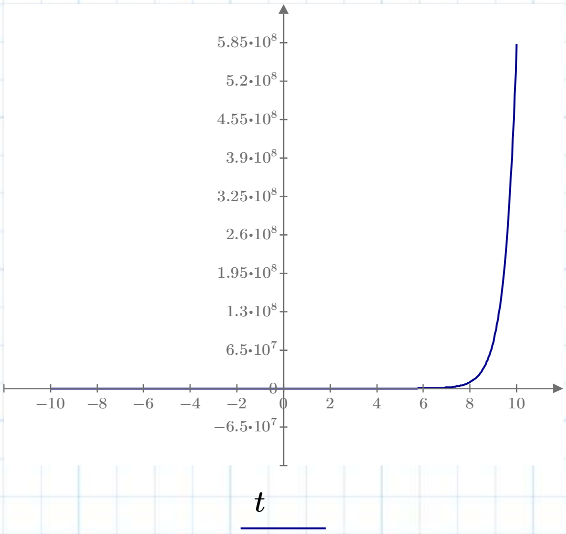
$$\left(s^2 \cdot Y(s) - s \cdot 0 - 1\right) - 3 \cdot \left(s \cdot Y(s) - 0\right) + 2 \cdot Y(s) = \frac{1}{s^2+1}$$

$$s^2 \cdot Y(s) - 1 - 3 \cdot s \cdot Y(s) + 2 \cdot Y(s) = \frac{1}{s^2+1}$$

$$\left(s^2 - 3 \cdot s + 2\right) \cdot Y(s) = \frac{s^2+2}{s^2+1}$$

$$Y(s) = \frac{s^2+2}{\left(s^2+1\right) \cdot \left(s^2 - 3 \cdot s + 2\right)}$$

$$\frac{s^2+2}{\left(s^2+1\right) \cdot \left(s^2 - 3 \cdot s + 2\right)} \xrightarrow{invlaplace} \frac{6 \cdot e^{2 \cdot t}}{5} + \frac{3 \cdot \cos(t)}{10} - \frac{3 \cdot e^{t}}{2} + \frac{\sin(t)}{10}$$



$$\frac{6 \cdot e^{2 \cdot t}}{5} + \frac{3 \cdot \cos(t)}{10} - \frac{3 \cdot e^{t}}{2} + \frac{\sin(t)}{10}$$

② $\dfrac{dy}{dx} - y = x^2 \quad x = 1, y = 1$

Q.14-3　上記の微分方程式を解きグラフ表示せよ。

推定値

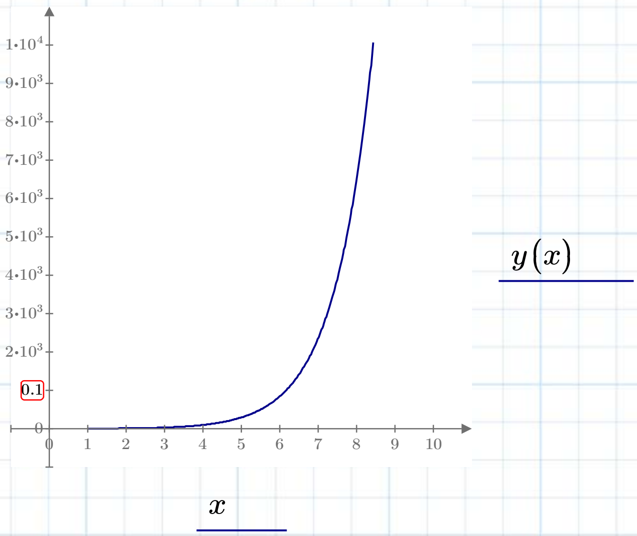
$y(x) := 0$

制約条件

$\dfrac{\mathrm{d}}{\mathrm{d}x} y(x) - y(x) = \mathrm{x}^2$　　$y(1) = 1$

ソルバー

$y := \mathbf{odesolve}(y(x), 10)$



Q.14-4　上記の微分方程式をLaplace変換を用いて解きグラフ表示せよ。

$$s \cdot Y(s) - y(0) - Y(s) = \frac{2}{s^3} \qquad t^2 \xrightarrow{laplace} \frac{2}{s^3}$$

$$(s-1)\, Y(s) = \frac{2}{s^3} + y(0)$$

$$Y(s) = \frac{2 + y_0 \cdot s^3}{s^3 \cdot (s-1)} \qquad \frac{2 + y_0 \cdot s^3}{s^3 \cdot (s-1)} \xrightarrow{invlaplace} 2 \cdot e^t - 2 \cdot t - t^2 + y_0 \cdot e^t - 2$$

$$y(t) := \left(2 \cdot e^t - 2 \cdot t - t^2 + y_0 \cdot e^t - 2\right)$$

$$y(1) \rightarrow 2 \cdot e + y_0 \cdot e - 5$$

$$\left(2 \cdot e + y_0 \cdot e - 5\right) = 1 \xrightarrow{solve, y_0} 6 \cdot e^{-1} - 2$$

$$\left(2 + y_0\right) \cdot e = 6$$

$$\left(2 + y_0\right) = 6 \cdot e^{-1}$$

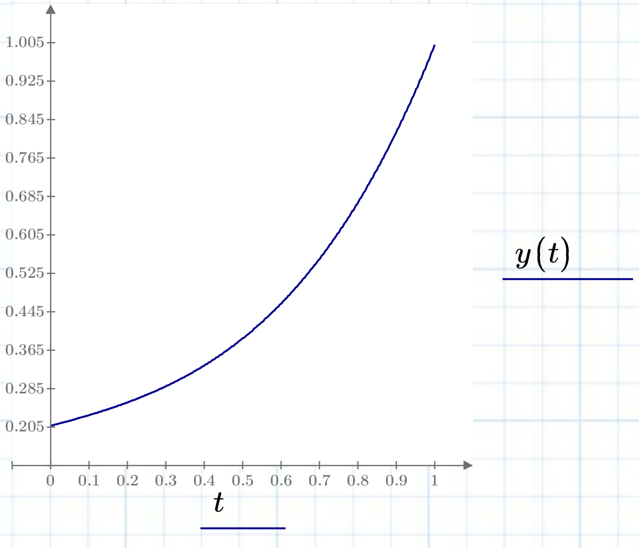
$$y_0 = 6 \cdot e^{-1} - 2$$

$$y_0 := 6 \cdot e^{-1} - 2$$

$$y(t) := \left(2 \cdot e^t - 2 \cdot t - t^2 + y_0 \cdot e^t - 2\right)$$

$$y(1) = 1$$

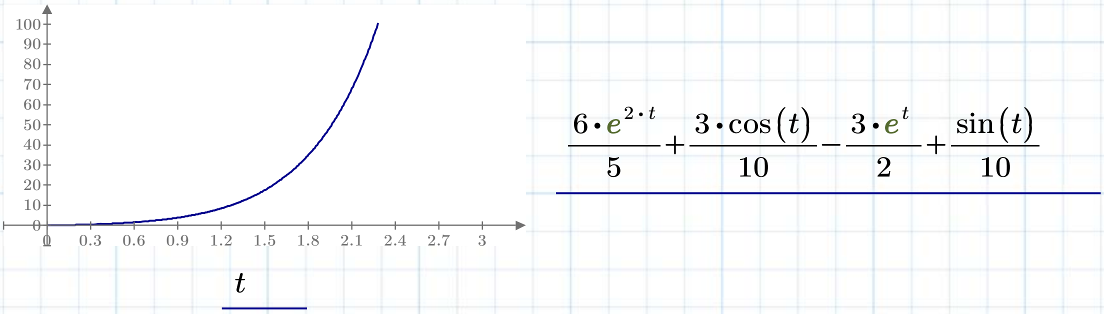
$$y(t) := \left(2 \cdot e^t - 2 \cdot t - t^2 + \left(6 \cdot e^{-1} - 2\right) \cdot e^t - 2\right)$$



Q.14-5　上記の微分方程式をLaplace変換を用いて解きグラフ表示せよ。　$\text{clear}(y)$

$$\left(s^2 \cdot Y(s) - s \cdot y(0) - y'(0)\right) - 3 \cdot \left(s \cdot Y(s) - y(0)\right) + 2 \cdot Y(s) = \frac{1}{s^2+1} \xrightarrow[\substack{\text{substitute}, y(0)=0 \\ \text{substitute}, y'(0)=1 \\ \text{solve}, Y(s) \\ \text{invlaplace}}]{} \frac{6 \cdot e^{2 \cdot t}}{5} + \frac{3 \cdot \cos(t)}{10} - \frac{3 \cdot e^{t}}{2} + \frac{\sin(t)}{10}$$

$$\left(s^2 \cdot Y(s) - s \cdot y(0) - y'(0)\right) - 3 \cdot \left(s \cdot Y(s) - y(0)\right) + 2 \cdot Y(s) = \frac{1}{s^2+1} \xrightarrow[\substack{\text{solve}, Y(s) \\ \text{invlaplace} \\ \text{substitute}, y(0)=0 \\ \text{substitute}, y'(0)=1}]{} \frac{6 \cdot e^{2 \cdot t}}{5} + \frac{3 \cdot \cos(t)}{10} - \frac{3 \cdot e^{t}}{2} + \frac{\sin(t)}{10}$$



$\text{clear}(y, t, s, y0)$

$$s \cdot Y(s) - y(0) - Y(s) = \frac{2}{s^3} \xrightarrow[\substack{\text{solve}, Y(s) \\ \text{invlaplace}}]{} 2 \cdot e^{t} - 2 \cdot t - t^2 + e^{t} \cdot y(0) - 2$$

$$1 = 2 \cdot e^{t} - 2 \cdot t - t^2 + e^{t} \cdot y0 - 2 \xrightarrow[\substack{\text{substitute}, t=1 \\ \text{solve}, y0}]{} 6 \cdot e^{-1} - 2$$

$$y(t) := 2 \cdot e^{t} - 2 \cdot t - t^2 + e^{t} \cdot y0 - 2 \xrightarrow{\text{substitute}, y0 = 6 \cdot e^{-1} - 2} 6 \cdot e^{t-1} - 2 \cdot t - t^2 - 2$$