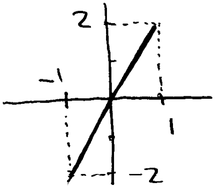
2.3 5. $f(x)=2x \quad (-1<x<1) \quad p=2L=2$ の フーリエ級数



$\left.\begin{array}{l} f(x) \\ f(x)\cos\dfrac{kx\pi}{L} \end{array}\right\}$ 奇

$f(x)\sin\dfrac{k\pi x}{L}$ 偶

$p=2L=2 \qquad L=1$

$$a_k=\frac{1}{1}\int_{-1}^{1} f(x)\cos\frac{k\pi x}{1}\,dx=0$$

$$b_k=\frac{2}{1}\int_{0}^{1} 2x\sin\frac{k\pi x}{1}\,dx$$

$$=4\left(\left[x\frac{-1}{k\pi}\cos\frac{k\pi x}{1}\right]_0^1-\int_0^1 1\cdot\frac{-1}{k\pi}\cos\frac{k\pi x}{1}\,dx\right)$$

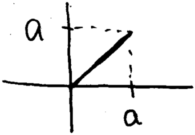
$$=4\left[\frac{-x}{k\pi}\cos\frac{k\pi x}{1}+\frac{1}{(k\pi)^2}\sin\frac{k\pi x}{1}\right]_0^1$$

$$=4\left(\frac{-1}{k\pi}\cos k\pi+\frac{1}{(k\pi)^2}\sin k\pi\right)$$

$$=\frac{-4}{k\pi}\cos k\pi=\frac{-4}{k\pi}(-1)^k$$

$$f(x)=\sum_{k=1}^{\infty}\frac{-4}{k\pi}(-1)^k\sin\frac{k\pi x}{1}$$

$$=\frac{4}{\pi}\left(\sin\pi x-\frac{1}{2}\sin 2\pi x+\frac{1}{3}\sin 3\pi x-\cdots\right)$$



2.8.9 $f(x) = \begin{cases} x & (0<x<a) \\ 0 & (x>a) \end{cases}$

のフーリエ余弦積分表示

$$C(\alpha) = \frac{2}{\pi}\int_0^\infty f(t)\cos\alpha t\,dt$$

$$= \frac{2}{\pi}\left(\int_0^a x\cos\alpha t\,dt + \overbrace{\int_a^\infty 0\,dt}^{=0}\right)$$

$$= \frac{2}{\pi}\left(\left[x\frac{\sin\alpha t}{\alpha}\right]_0^a - \int_0^a 1\cdot\frac{\sin\alpha t}{\alpha}dt\right)$$

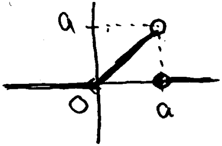
$$= \frac{2}{\pi}\left(\left[x\frac{\sin\alpha t}{\alpha} + \frac{\cos\alpha t}{\alpha^2}\right]_0^a\right)$$

$$= \frac{2}{\pi}\left(a\frac{\sin\alpha a}{\alpha} + \frac{\cos\alpha a}{\alpha^2} - \frac{1}{\alpha^2}\right)$$

$$f(x) = \int_0^\infty \frac{2}{\pi}\left(a\frac{\sin\alpha a}{\alpha} + \frac{\cos\alpha a - 1}{\alpha^2}\right)\cos\alpha x\,d\alpha$$

2.10

5. $f(x)=\begin{cases} x & (0<x<a) \\ 0 & (\text{それ以外}) \end{cases}$ のフーリエ変換



$$\hat{f}(\alpha)=\frac{1}{\sqrt{2\pi}}\left(\int_{-\infty}^{0} 0\,dx+\int_{0}^{a} x e^{-i\alpha x}\,dx+\int_{a}^{\infty} 0\,dx\right)$$

$$=\frac{1}{\sqrt{2\pi}}\left(\left[x\frac{1}{-i\alpha}e^{-i\alpha x}\right]_0^a-\int_0^a 1\cdot\frac{1}{-i\alpha}e^{-i\alpha x}\,dx\right)$$

$$=\frac{1}{\sqrt{2\pi}}\left(\left[x\frac{1}{-i\alpha}e^{-i\alpha x}-\frac{1}{(-i\alpha)^2}e^{-i\alpha x}\right]_0^a\right)$$

$$=\frac{1}{\sqrt{2\pi}}\left(\frac{a}{-i\alpha}e^{-i\alpha a}+\frac{1}{\alpha^2}e^{-i\alpha a}-\frac{1}{\alpha^2}\right)$$