複素数 (complex number) の定義　　複素数とは？

[p.116]

$\alpha = a + ib = a + bi$　　　　$\alpha = Re(\alpha) + iIm(\alpha)$

$i = \sqrt{-1}$　虚数単位 (imaginary unit)　　　$i^2 = -1$

$a = Re(\alpha)$　実部 (real part)

$b = Im(\alpha)$　虚部 (imaginary part)

a, b は　実数 (real number)

[p.116]

$\alpha = a = a + i0$ は　実数 (real number)　　$Im(\alpha) = 0$

$\alpha = ib = 0 + ib$ は　純虚数 (pure imaginary number)　　$Re(\alpha) = 0$

$0 = 0 + i0$

[p.116]

$\alpha = a + ib,\quad \beta = c + id$ の計算

相等 $\alpha = \beta \Leftrightarrow \begin{cases} a = c & \text{実部} \\ b = d & \text{虚部} \end{cases}$　とくに，$\alpha = 0 \Leftrightarrow \begin{cases} a = 0 \\ b = 0 \end{cases}$

四則演算

$$\alpha + \beta = (a+ib) + (c+id) = a + ib + c + id = (a+c) + i(b+d)$$

$$\alpha - \beta = (a+ib) - (c+id) = a + ib - c - id = (a-c) + i(b-d)$$

$$\alpha\beta = (a+ib)(c+id) = ac + iad + ibc + i^2bd = (ac-bd) + i(ad+bc)$$

$$\frac{\alpha}{\beta} = \frac{a+ib}{c+id} = \frac{(a+ib)(c-id)}{(c+id)(c-id)} = \frac{ac - iad + ibc - i^2bd}{c^2 - i^2d^2} = \frac{ac+bd}{c^2+d^2} + i\frac{bc-ad}{c^2+d^2}$$

代数計算をし，$i^2 = -1$ を代入．実部と虚部に整理して，結果も複素数．

[p.116—p.117]

$Re(\alpha\beta) = ac - bd$　　　　$Im\left(\dfrac{\alpha}{\beta}\right) = \dfrac{bc-ad}{c^2+d^2}$

例 1 $\dfrac{4-2i}{3+i}=\dfrac{(4-2i)(3-i)}{(3+i)(3-i)}=\dfrac{12-4i-6i+2i^2}{9-i^2}=\dfrac{10-10i}{10}=1-i$ ∥

[p.117 例 1]

関連問題 [p.117 問 1][p.131 1.]

1 次の計算をせよ. (a) $(-2+7i)-(8-6i)=$ $-2+7i-8+6i=-10+13i$

(b) $(5-2i)(3+2i)=$ $15+10i-6i-4i^2=19+4i$

(c) $\dfrac{3+i}{1+i}=$ $\dfrac{(3+i)(1-i)}{(1+i)(1-i)}=\dfrac{3-3i+i-i^2}{1^2-i^2}=\dfrac{4}{2}+\dfrac{-2i}{2}=2-i$

(d) $i^3=$ $i^2\cdot i=-i$ $\quad i^4=$ $i^2\cdot i^2=1$ $\quad i^5=$ $i^2\cdot i^2\cdot i=i$

(e) $\dfrac{2i^5}{1+i^3}=$ $\dfrac{2i}{1-i}=\dfrac{2i(1+i)}{(1-i)(1+i)}=\dfrac{2i+2i^2}{1^2-i^2}=\dfrac{-2}{2}+\dfrac{2i}{2}=-1+i$

(f) $(2i^3+3i^5)(i^4-2i^5)=$ $(-2i+3i)(1-2i)=i(1-2i)=i-2i^2=2+i$

(g) $\left(\cos\dfrac{\pi}{9}+i\sin\dfrac{\pi}{9}\right)\left(\cos\dfrac{7\pi}{18}+i\sin\dfrac{7\pi}{18}\right)=$

$$=\left(\cos\frac{\pi}{9}\cos\frac{7\pi}{18}-\sin\frac{\pi}{9}\sin\frac{7\pi}{18}\right)+i\left(\sin\frac{\pi}{9}\cos\frac{7\pi}{18}+\cos\frac{\pi}{9}\sin\frac{7\pi}{18}\right)$$

$$=\cos\Big(\underbrace{\frac{\pi}{9}+\frac{7\pi}{18}}\Big)+i\sin\Big(\underbrace{\frac{\pi}{9}+\frac{7\pi}{18}}_{=\frac{9\pi}{18}=\frac{\pi}{2}}\Big)=\underbrace{\cos\frac{\pi}{2}}_{0}+\underbrace{i\sin\frac{\pi}{2}}_{1}=1$$

(h) $Re\left(\dfrac{3+i}{1+i}\right)=$ $Re(2-i)=2$ $\qquad$ (i) $Im\left(\dfrac{2i^5}{1+i^3}\right)=$ $Im(-1+i)=1$

$$(5+2i)(3-2i)=15-10i+6i-4i^2=19-4i$$

$$\frac{1+i}{3+i}=\frac{(1+i)(3-i)}{(3+i)(3-i)}=\frac{3-i+3i-i^2}{9-i^2}=\frac{4}{10}+\frac{2i}{10}=\frac{2}{5}+\frac{i}{5}$$

$$\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right)\div\left(\cos\frac{8\pi}{15}+i\sin\frac{8\pi}{15}\right)=\frac{\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right)\left(\cos\frac{8\pi}{15}-i\sin\frac{8\pi}{15}\right)}{\left(\cos\frac{8\pi}{15}+i\sin\frac{8\pi}{15}\right)\left(\cos\frac{8\pi}{15}-i\sin\frac{8\pi}{15}\right)}$$

$$=\frac{\left(\cos\frac{\pi}{5}\cos\frac{8\pi}{15}+\sin\frac{\pi}{5}\sin\frac{8\pi}{15}\right)+i\left(\sin\frac{\pi}{5}\cos\frac{8\pi}{15}-i\cos\frac{\pi}{5}\sin\frac{8\pi}{15}\right)}{\cos^2\frac{8\pi}{15}-i^2\sin^2\frac{8\pi}{15}}$$

$$=\cos\Big(\underbrace{\frac{\pi}{5}-\frac{8\pi}{15}}_{-\frac{5}{15}\pi=-\frac{\pi}{3}}\Big)+i\sin\left(\frac{\pi}{5}-\frac{8\pi}{15}\right)=\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)=\frac{\pi}{2}-i\frac{\sqrt{3}}{2}$$