共役複素数 (complex conjugate) $\overline{\alpha}$　　共役複素数とは？ [p.116]

$\alpha = a + ib$ に対して　　$\overline{\alpha} = a - ib$　　虚部の符号を変える [p.117—118]

$\overline{\overline{\alpha}} = \alpha$

$\alpha + \overline{\alpha} = (a+i\cancel{b})+(a-i\cancel{b}) = 2a = 2\,\mathrm{Re}(\alpha)$　　$\therefore\ \mathrm{Re}(\alpha) = \dfrac{\alpha+\overline{\alpha}}{2}$

$\alpha - \overline{\alpha} = (\cancel{a}+ib)-(\cancel{a}-ib) = 2ib = 2i\,\mathrm{Im}(\alpha)$　　$\therefore\ \mathrm{Im}(\alpha) = \dfrac{\alpha-\overline{\alpha}}{2i}$ [p.117]

$\overline{\alpha+\beta} = \overline{\alpha} + \overline{\beta}$

$\overline{\alpha-\beta} = \overline{\alpha} - \overline{\beta}$

$\overline{\alpha\beta} = \overline{\alpha}\,\overline{\beta}$

$\overline{\left(\dfrac{\alpha}{\beta}\right)} = \dfrac{\overline{\alpha}}{\overline{\beta}}$ [p.118]

関連問題 [p.117 問 2][p.118 問 3 問 4]

2a $\alpha = a + ib,\ \beta = c + id$ として,

(a) $\overline{\alpha\beta} = \overline{\alpha}\,\overline{\beta}$　$\because\ \overline{\alpha\beta} = \cdots = \overline{(ac-bd)+i(ad+bc)} = (ac-bd)-i(ad+bc)$

$\overline{\alpha}\,\overline{\beta} = (a-ib)(c-id) = ac-aid-ibc+i^2bd$

$= (ac-bd)-i(ad+bc)$

(b) $\alpha\overline{\beta} - \overline{\alpha}\beta = (a+ib)(c-id)-(a-ib)(c+id)$

$= (\cancel{ac}-aid+ibc-\cancel{i^2bd})-(\cancel{ac}+aid-ibc-\cancel{i^2bd}) = 2i(bc-ad)$ 純虚数

絶対値 (absolute value) $|\alpha|$　　絶対値とは？ [p.116]

$\alpha = a + ib$ に対して　　$|\alpha| = \sqrt{a^2+b^2}$　　$\sqrt{(\text{実部})^2+(\text{虚部})^2}$ [p.117—118]

実数a $|a| = \sqrt{a^2}$

$\alpha\overline{\alpha} = (a+ib)(a-ib) = a^2-i^2b^2$　　$\therefore\ |\alpha|^2 = \alpha\overline{\alpha}$　　$\therefore\ |\alpha| = \sqrt{\alpha\overline{\alpha}}$

$= a^2+b^2$

$|\alpha| = 0$　$\sqrt{a^2+b^2} = 0$　　$\therefore\ a=0,\ b=0$　　$\therefore\ \alpha = 0$ [p.117]

$|\alpha+\beta| \leqq |\alpha|+|\beta|$

$|\alpha-\beta| \geqq \big||\alpha|-|\beta|\big| \geqq |\alpha|-|\beta|$

$|\alpha\beta| = |\alpha||\beta|$

$\left|\dfrac{\alpha}{\beta}\right| = \dfrac{|\alpha|}{|\beta|}$

関連問題 [p.117 問 2][p.118 問 3 問 4]

2b $\alpha = a + ib,\ \beta = c + id$ として,

(a) $\left|\frac{\alpha}{\beta}\right| = \frac{|\alpha|}{|\beta|} \quad \because \left|\frac{\alpha}{\beta}\right| = \cdots = \left|\frac{ac+bd}{c^2+d^2} + i\frac{bc-ad}{c^2+d^2}\right| = \sqrt{\left(\frac{ac+bd}{c^2+d^2}\right)^2 + \left(\frac{bc-ad}{c^2+d^2}\right)^2}$

$= \sqrt{\frac{a^2c^2+\cancel{2acbd}+b^2d^2+b^2c^2-\cancel{2bcad}+a^2d^2}{(c^2+d^2)^2}} = \sqrt{\frac{a^2c^2+b^2d^2+b^2c^2+a^2d^2}{(c^2+d^2)^2}}$

$\frac{|\alpha|}{|\beta|} = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}} = \sqrt{\frac{a^2+b^2}{c^2+d^2}} = \sqrt{\frac{(a^2+b^2)(c^2+d^2)}{(c^2+d^2)^2}} = \sqrt{\frac{a^2c^2+a^2d^2+b^2c^2+b^2d^2}{(c^2+d^2)^2}}$

(b) $|Re(\alpha)| \leqq |\alpha| \quad \because |Re(\alpha)| = |a| = \sqrt{a^2} \qquad |\alpha| = \sqrt{a^2+b^2}$

$b^2 \geqq 0$ だから

(c) $|\alpha - \beta| \geqq ||\alpha| - |\beta|| \quad \because |\alpha-\beta|^2 = \cdots = |(a-c)+i(b-d)|^2$

$= (a-c)^2 + (b-d)^2$

$= a^2+b^2+c^2+d^2-2(ac+bd)$

$(\geqq a^2+b^2+c^2+d^2-2\sqrt{(ac+bd)^2})$

$||\alpha|-|\beta||^2 = \left(\sqrt{a^2+b^2} - \sqrt{c^2+d^2}\right)^2$

$= a^2+b^2+c^2+d^2-2\sqrt{(a^2+b^2)(c^2+d^2)}$

$= a^2+b^2+c^2+d^2-2\sqrt{a^2c^2+a^2d^2+b^2c^2+b^2d^2}$

$= a^2+b^2+c^2+d^2-2\sqrt{(ac+bd)^2+(ad-bc)^2} \leqq |\alpha-\beta|^2$

$|\alpha\beta| = |\alpha||\beta|$

$\because |\alpha\beta| = |(a+ib)(c+id)| = |ac+aid+ibc+i^2bd|$

$= |(ac-bd)+i(ad+bc)| = \sqrt{(ac-bd)^2+(ad+bc)^2}$

$= \sqrt{a^2c^2-\cancel{2acbd}+b^2d^2+a^2d^2+\cancel{2adbc}+b^2c^2} = \sqrt{a^2c^2+b^2d^2+a^2d^2+b^2c^2}$

$|\alpha||\beta| = \sqrt{a^2+b^2}\sqrt{c^2+d^2} = \sqrt{(a^2+b^2)(c^2+d^2)} = \sqrt{a^2c^2+a^2d^2+b^2c^2+b^2d^2}$

$\bar{\alpha}\beta + \alpha\bar{\beta} = (a-ib)(c+id) + (a+ib)(c-id)$

$= (ac+\cancel{aid}-\cancel{ibc}-i^2bd) + (ac-\cancel{aid}+\cancel{ibc}-i^2bd)$

$= 2(ac+bd)$ 実数

$|Im(\alpha)| \leqq |\alpha|$

$|Im(\alpha)| = |b| = \sqrt{b^2}$

$|\alpha| = \sqrt{a^2+b^2} \qquad a^2 \geqq 0$ だから