Department of Electronic Control Engineering

Numerical solutions for reduced nonlinear differential equations of physical fluid

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Research Outline

Numerical solutions for reduced nonlinear differential equations of physical fluid

Physical fluid is described by equations of motion , a system of nonlinear partial differential equations for physical quantities, e.g. velocity, pressure, magnetic field, and temperature.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{1}{\rho_0}\nabla p - \frac{\rho}{\rho_0} g\hat{\mathbf{z}} + \frac{1}{\mu_0\rho_0}(\nabla\times\mathbf{B})\times\mathbf{B} + \nu\nabla^2\mathbf{u}, \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + \eta\nabla^2\mathbf{B}, \quad \nabla\cdot\mathbf{B} = 0, \tag{2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T = \kappa\nabla^2 T, \tag{3}$$

Perturbed quantities, e.g. stream function and flux function,

$$\mathbf{u} = \left(-\frac{\partial\phi}{\partial z}, 0, \frac{\partial\phi}{\partial x}\right), \tag{4}$$

$$\mathbf{B} = \left(-\frac{\partial A}{\partial z}, 0, \frac{\partial A}{\partial x}\right), \tag{5}$$

and some conditions make this system reduced as follows;

$$\frac{1}{\sigma}\left(\frac{\partial\nabla^2\phi}{\partial t} + \{\phi, \nabla^2\phi\}\right) = R\frac{\partial\theta}{\partial x} + \nabla^4\phi + \zeta Q\left(\frac{\partial}{\partial z}\nabla^2 A + \{A, \nabla^2 A\}\right), \tag{11}$$

$$\frac{\partial\theta}{\partial t} + \{\phi, \theta\} = \nabla^2\theta + \frac{\partial\phi}{\partial x}, \tag{12}$$

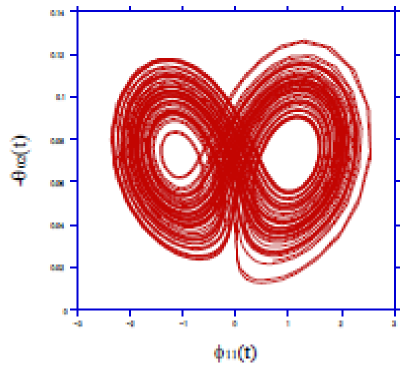
$$\frac{\partial A}{\partial t} + \{\phi, A\} = \zeta\nabla^2 A + \frac{\partial\phi}{\partial z}, \tag{13}$$

where $\theta(x,z,t) = T - (1-z)$.

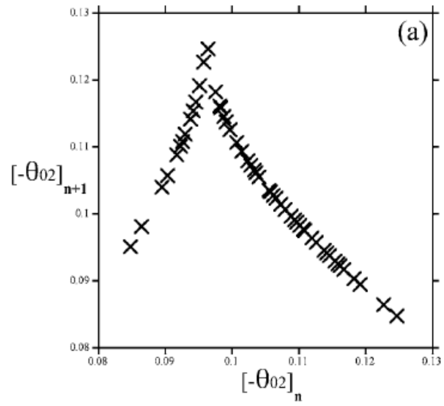
Some chaotic behavior often occurs in such deterministic system. My research is for some parameters coherent structures (like solitary waves) are formed in this chaotic motion.

Some lower components of perturbed quantities in Fourier sine series and Fourier cosine series shows evidence that chaotic behavior happened.

Phase portrait:



Poincare return map;



Coherent structure appears in irregular motions in a phase portrait. This structure may be related to this fluid control.