Words and Diophantine Approximation

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Research Outline

Research on words

A sequence of letters is called a word. A word is relate to Diophantine approximation.

A number of subwords with length n is called complexity p(n). If a word is periodic then complexity is bounded. The Sturmian words is the non periodic words which have least complexity p(n)=n+1.

The Kolakoski sequence is a sequence which satisfies the condition that the sequence of its runlengths is equal to itself.

1 2 2 1 1 2 1 2 2 1 2 2 1 1 2 1 1 2 2 1 2
1 2 2 1 1 2 1 2 2 1 2 2 1 1

Kolakoski sequence {1,2}

The frequency of 1 in Kolakoski sequence {1,2} is conjectured 0.5. And the complexity of Kolakoski sequence {1,2} is conjectured p(n) ~ $n^{\log 3/\log(3/2)}$.

Kolaloski sequence {1,3} is the word

1 3 3 3 1 1 1 3 3 3 1 3 1 3 3 3 1 1 1 3 3 3 .
1 3 3 3 1 1 1 3 3 3

The frequency of 1 in Kolakoski sequence {1,3} is a root of a equation

$$4x^3-14x^2+15x-4=0$$

I prove the complexity of Kolakoski sequence {1,3} p(n)=2n+2 for large n. I define a restricted complexity p_odd(n) and p_even(n), and satisfies p_odd(n)=p_even(n)=n+1 for Kolakoski sequence{1,3}. Kolakoski sequence {1,3} is similar to Sturmian words.

1444477771111444477777771111111444444477777777 1 4 7 1 4444 7777
1 4 4 4 4 7 7 7 7 1 1 1 1 4 4

This word is Kolakoski sequence {1,4,7}. The frequency of 1 is a root of cubic equation

$$36x^3-84x^2+60x-11=0 \ .$$