Construction of Field Theory on a Graph

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● Research Outline

We study various models on dimensional deconstruction (*1) and construct field theory on a graph to generalize dimensional deconstruction.

Dimensional deconstruction (DD) is a higher-dimensional gauge theory with discretized extra dimensions. This model is described by a ``moose" diagram (Fig. 1), where gauge groups are represented by open circles, and non-linear sigma model fields by single directed lines attached to these circles.
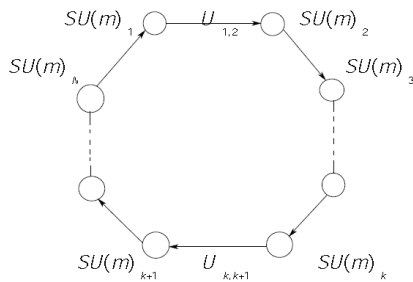

Fig. 1. Moose diagram

If we assume that each non-linear sigma model field has a same value of vacuum expectation, we obtain a gauge boson mass spectrum after a mass matrix diagonalization. In the limit of short length of the lattice spacing, the spectrum is almost equal to the Kaluza-Klein mass spectrum for a five-dimensional gauge boson compactified on a circle. This indicates that the conceptual moose diagram shows the latticized fifth dimension, which is physical space.

If we consider a higher-dimensional deconstruction, then open circles may be complicatedly connected by single lines. Such a connection is a graph. In order to generalize DD, we have constructed field theory on a graph.

There are several representation matrices associated with a graph. Using these matrices and the knowledge of spectral graph theory, we have shown that mass matrices for fields assigned to vertices and to edges are correspond to graph Laplacian and to the transpose matrix of it, respectively. We have also shown that the effective potential for the constant background link field is related to these matrices, and it is finite at the one-loop level. These facts enables us to construct the divergence-controlled models.

We have considered various models based on graph field theory, such as supersymmetric model, Nambu-Jona-Lasinio model, kink-like configurations (Fig. 2), and others.
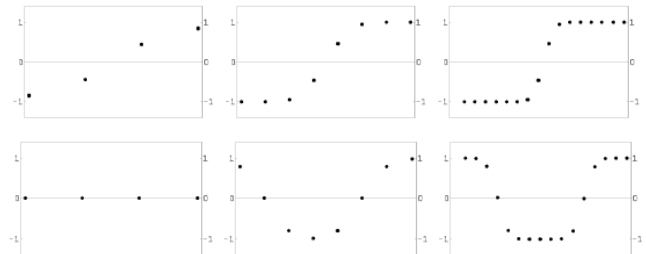

Fig. 2. Numerical solutions for the scalar field configuration in the case with Path graph in the first row, and with Cycle graph in the second.

We have also considered several models of induced gravity with the help of knowledge in spectral graph theory. Because the UV divergences can be controlled by the graph Laplacian, the one-loop finite Newton's constant is induced and the positive-definite cosmological constant can be obtained. We have also studied Einstein universe in induce gravity model. Self-consistent solutions can be found by proper choice of filed contents and graphs (Fig. 3).
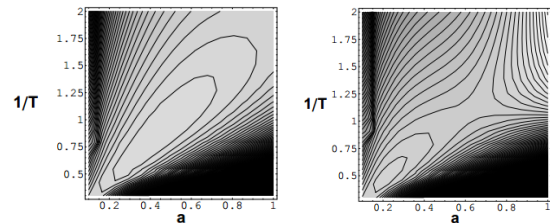

Fig. 3. A contour plot of βF in the two models. Solutions of the self-consistent equation can be found at the maximum and the saddle point.

We will continue to study field theory with graph structure and apply the results to various models.

Note 1: Dimensional deconstruction have been proposed by N. Arkani-Hamed, A. G. Cohen and H. Georgi, and the C. T. Hill, S. Pokorski, J. Wang and H.-C. Cheng, independently.