Mordell-Weil lattices of fibred rational surfaces and its applications to singularities

Shinya Kitagawa
Associate Professor, Ph.D. in Science
Email : kit058shiny@gifu-nct.ac.jp

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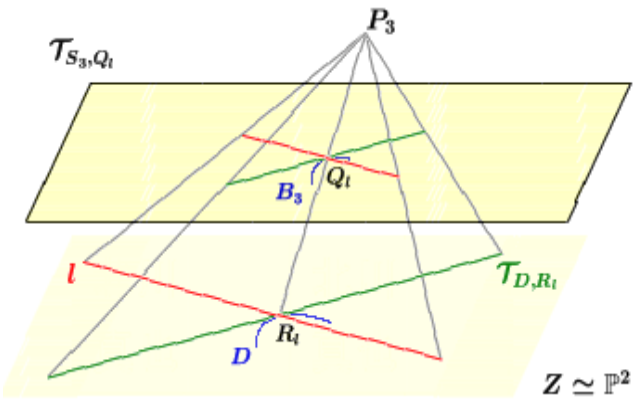
Research Outline

Extremal hyperelliptic fibrations on rational surfaces

The theory of the Mordell-Weil lattices are sufficiently developed by Oguiso and Shioda for minimal elliptic rational surfaces. In their work, the even unimodular root lattice E_8 of rank eight played very important role as the predominant frame. For example, it was shown that the Mordell-Weil group is trivial if and only if there exists a singular fibre of type II^* in the sense of Kodaira whose dual graph contains E_8 as a subgraph. The lattice E_8 also appears in another application by Shioda to describe a hierarchy of deformations of rational double points.

Let X be a smooth projective rational surface and f:X→P^1 a relatively minimal fibration whose general fibre is a projective curve of genus g>1. We know the Picard number ρ(X) is less than or equal to 4g+6, and consider the case ρ(X)=4g+6. Then the maximal Mordell-Weil lattice is isomorphic to the unimodular lattice called D^+_{4g+4} of rank 4g+4. Furthermore, Saito gives an example of f:X→P^1 whose Mordell-Weil group is trivial and which has an extension of a singular fibre of type II^*. Since D^+_8=E_8, we expect an application similar to the elliptic case.



Mordell-Weil lattices of genus two fibrations of degree nine on rational surfaces

We consider a smooth rational surface together with a relatively minimal fibration of genus two. Assume that the Picard number of the surface equals twelve and the fibration has a section whose self-intersection number is minus one. Then the surface admits an elliptic fibration whose Mordell-Weil group as well as lattice are isomorphic to those of the original fibration of genus two.