Singular Limit of the Case of Non-equal Deinsities for Incompressible Two Phase Flows with Phase Transitions

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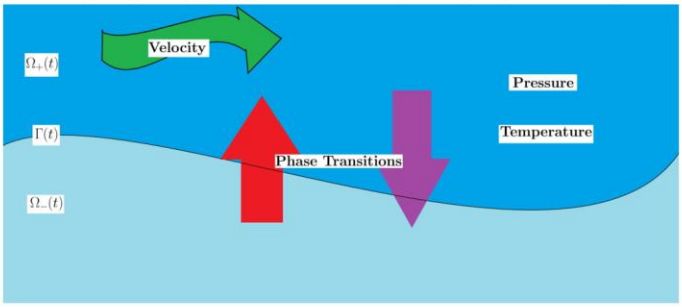
Research Outline

We describe the problem of incompressible two phase flows with phase transition and issues in this problem. Set

$$\Gamma(t) = \{(x', x_n) \in \mathbb{R}^{n-1} \times \mathbb{R} : \; x_n - h(t, x') = 0, \; t \geq 0\}$$

and

$$\Omega_\pm(t) = \{x = (x', x_n) \in \mathbb{R}^{n-1} \times \mathbb{R} : \; \pm(x_n - h(t, x')) > 0, \; t \geq 0\}.$$



Flows with phase transitions is composed of problems in fluid mechanics and thermodynamics, so we write this problem like the following which consists of next three parts:

$$\begin{aligned}
\rho(\partial_t u + u \cdot \nabla u) - \operatorname{div} T(u, \pi, \theta) &= 0 && \text{in} \quad \Omega(t), \, t > 0, \\
\operatorname{div} u &= 0 && \text{in} \quad \Omega(t), \, t > 0, \\
[\![\frac{1}{\rho}]\!] j^2 \nu_\Gamma - [\![T(u, \pi, \theta)\nu_\Gamma]\!] - \sigma H_\Gamma \nu_\Gamma &= 0 && \text{on} \quad \Gamma(t), \, t > 0, \\
[\![u]\!] - [\![\frac{1}{\rho}]\!] j \nu_\Gamma &= 0 && \text{on} \quad \Gamma(t), \, t > 0, \\
u(0) &= u_0 && \text{in} \quad \Omega(t),
\end{aligned}$$

$$\begin{aligned}
\rho\kappa(\theta)(\partial_t \theta + u \cdot \nabla\theta) - \operatorname{div}(d(\theta)\nabla\theta) - 2\mu(\theta)|D(u)|_2^2 &= 0 && \text{in} \quad \Omega(t), \, t > 0, \\
l(\theta) j + [\![d(\theta)\partial_{\nu_\Gamma}\theta]\!] &= 0 && \text{on} \quad \Gamma(t), \, t > 0, \\
[\![\theta]\!] &= 0 && \text{on} \quad \Gamma(t), \, t > 0, \\
\theta(0) &= \theta_0 && \text{in} \quad \mathbb{R}^n,
\end{aligned}$$

$$\begin{aligned}
[\![\psi(\theta)]\!] + [\![\frac{1}{2\rho^2}]\!] j^2 - [\![\frac{T(u, \pi, \theta)\nu_\Gamma \cdot \nu_\Gamma}{\rho}]\!] &= 0 && \text{on} \quad \Gamma(t), \, t > 0, \\
V_\Gamma - u \cdot \nu_\Gamma + \frac{1}{\rho} j &= 0 && \text{on} \quad \Gamma(t), \, t > 0, \\
\Gamma(0) &= \Gamma_0.
\end{aligned}$$

We treat incompressible two phase flows with phase transition in the case of equal densities as singular limits of the case of non-equal densities, where "singular limits" means that regularity of the solution is continuous for the limit $\rho_+ \to \rho_-$.

In the case of non-equal densities, there exists a positive number T and the solution, h has regularity:

$$h \in W_p^2(J; L_q(\mathbb{R}^n)) \cap W_p^1(J; W_q^2(\dot{\mathbb{R}}^n)) \cap L_p(J; W_q^3(\dot{\mathbb{R}}^n)),$$

where $J = (0, T]$. So, we could see regularity of the height function in the case of non-equal densities is isotropic in $\mathbb{R}^n$ in a sense. However, the case of equal densities essentially consists of incompressible Navier-Stokes equation and two phase Stefan problem, so we should analyse regularity in this case carefully.