



Singular Limit of the Case of Non-equal Densities for Incompressible Two Phase Flows with Phase Transitions

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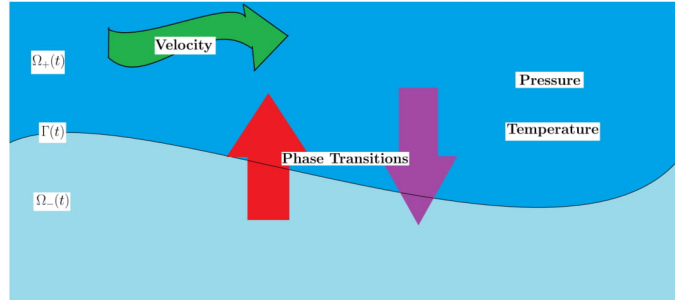
● Research Outline

We describe the problem of incompressible two phase flows with phase transition and issues in this problem. Set

$$\Gamma(t) = \{(x', x_n) \in \mathbb{R}^{n-1} \times \mathbb{R} : x_n - h(t, x') = 0, t \geq 0\}$$

and

$$\Omega_{\pm}(t) = \{x = (x', x_n) \in \mathbb{R}^{n-1} \times \mathbb{R} : \pm(x_n - h(t, x')) > 0, t \geq 0\}.$$



Flows with phase transitions is composed of problems in fluid mechanics and thermodynamics, so we write this problem like the following which consists of next three parts:

$$\begin{aligned} \rho(\partial_t u + u \cdot \nabla u) - \operatorname{div} T(u, \pi, \theta) &= 0 & \text{in } \Omega(t), t > 0, \\ \operatorname{div} u &= 0 & \text{in } \Omega(t), t > 0, \\ \left[\frac{1}{\rho}\right] j^2 \nu_{\Gamma} - \left[\frac{T(u, \pi, \theta) \nu_{\Gamma}}{\rho} \right] - \sigma H_{\Gamma} \nu_{\Gamma} &= 0 & \text{on } \Gamma(t), t > 0, \\ [u] - \left[\frac{1}{\rho}\right] j \nu_{\Gamma} &= 0 & \text{on } \Gamma(t), t > 0, \\ u(0) &= u_0 & \text{in } \Omega(t), \end{aligned}$$

$$\begin{aligned} \rho \kappa(\theta)(\partial_t \theta + u \cdot \nabla \theta) - \operatorname{div}(d(\theta) \nabla \theta) - 2\mu(\theta) |D(u)|_2^2 &= 0 & \text{in } \Omega(t), t > 0, \\ l(\theta) j + [d(\theta) \partial_{\nu_{\Gamma}} \theta] &= 0 & \text{on } \Gamma(t), t > 0, \\ [\theta] &= 0 & \text{on } \Gamma(t), t > 0, \\ \theta(0) &= \theta_0 & \text{in } \mathbb{R}^n, \end{aligned}$$

$$\begin{aligned} [\psi(\theta)] + \left[\frac{1}{2\rho^2}\right] j^2 - \left[\frac{T(u, \pi, \theta) \nu_{\Gamma} \cdot \nu_{\Gamma}}{\rho} \right] &= 0 & \text{on } \Gamma(t), t > 0, \\ V_{\Gamma} - u \cdot \nu_{\Gamma} + \frac{1}{\rho} j &= 0 & \text{on } \Gamma(t), t > 0, \\ \Gamma(0) &= \Gamma_0. \end{aligned}$$

We treat incompressible two phase flows with phase transition in the case of equal densities as singular limits of the case of non-equal densities, where "singular limits" means that regularity of the solution is continuous for the limit $\rho_+ \rightarrow \rho_-$.

In the case of non-equal densities, there exists a positive number T and the solution, h has regularity:

$$h \in W_p^2(J; L_q(\mathbb{R}^n)) \cap W_p^1(J; W_q^2(\mathbb{R}^n)) \cap L_p(J; W_q^3(\mathbb{R}^n)),$$

where $J = (0, T]$. So, we could see regularity of the height function in the case of non-equal densities is isotropic in \mathbb{R}^n in a sense. However, the case of equal densities essentially consists of incompressible Navier-Stokes equation and two phase Stefan problem, so we should analyse regularity in this case carefully.