Asymptotic behavior of small amplitude solutions to two-component system of nonlinear Schrödinger equations

Yuji SAGAWA
Lecturer, Dr. Sci.
Email : sagawa@gifu-nct.ac.jp

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Research Outline

Two-component system of nonlinear Schrödinger equations

we investigate initial value problem for a two-component system of cubic nonlinear Schrödinger equation:

$$\left(i\partial_t + \frac{1}{2}\partial_x^2\right)u = ai|u|^2u + bi|v|^2u, \quad \left(i\partial_t + \frac{1}{2}\partial_x^2\right)v = ci|u|^2v + di|v|^2v, \qquad t > 0, x \in \mathbf{R},$$

with the initial condition

$$u(0,x) = \varepsilon\varphi(x), v(0,x) = \varepsilon\psi(x), \qquad x \in \mathbf{R}$$

The coefficients a, b, c, d are non-zero real number. As well understood, cubic nonlinearity clarifies a critical situation when considering large time behavior of small solutions to nonlinear Schrödinger equations in one space dimension. In general, cubic nonlinearity should be regarded as a long-range perturbation. So this system has not only physical interest but also mathematical interest. The aim of this research is to classify large time behavior of u(t,x) and v(t,x) by the coefficients a, b, c, d. Without loss of generality, we may take a, d=1 or –1. The summary is as follows:

① a=1 or d=1
⇒ Finite time blow-up occurs(Kita, preprint)
② a=d=-1, b<0 or c<0
⇒ L^2-decay occurs(Li-Sunagawa'16)
③ a=d=-1, b>0, c>0, bc>1
⇒ Finite time blow-up occurs(Kita, preprint)
④ a=d=-1, b>0, c>0, bc=1
⇒ Sagawa(submitted) clarified large time behavior of solutions.
⑤ a=d=-1, b>0, c>0, bc<1
⇒ L^2-decay occurs(Li-Sunagawa'16)

Analysis.

To clarify large time behavior, we are going to reduce nonlinear Schrödinger system, based on the idea stemming from the previous work by Hayashi–Naumkin'98. We define

$$\alpha = \alpha(t,\xi) = \mathcal{F}[\mathrm{U}(-t)u(t\cdot)](\xi), \ \beta = \beta(t,\xi) = \mathcal{F}[\mathrm{U}(-t)v(t\cdot)](\xi)$$

We note that F denotes the Fourier transform and U(t) denotes the free Schrödinger operator. Then we can reduce nonlinear Schrödinger system to the following ordinary differential system:

$$i\partial_t\alpha = \frac{-i}{t}|\alpha|^2\alpha + \frac{bi}{t}|\beta|^2\alpha + R_1, \quad i\partial_t\beta = \frac{i}{bt}|\alpha|^2\beta - \frac{i}{t}|\beta|^2\beta + R_2, \qquad t > 1, \xi \in \mathbf{R},$$

where R_1 and R_2 can be regarded as remainder term. Asymptotic behavior of α(t,ξ) and β(t,ξ) reveals that of u(t,x) and v(t,x).